Course Assignment Spring 2013

EC3313 Industrial Economics

Sotiris Georganas

1) Long question (18 points)

Consider a market with two firms that produce goods which are differentiated with respect to their environmental friendliness: firm G produces a "Green" product of quality G, whereas firm B produces a "dirty" product of quality B. The marginal cost per unit of firm G is equal to c while the marginal cost of firm B is 0. Assume that $c > 0$ and $G > B > 0$. There is a continuum of consumers. Each consumer wishes to buy at most one unit (either G or B or none). Consumers differ with respect to their concern for the environment which is captured by the parameter a. The utility of a consumer whose taste parameter is a is given by

$$U_G = aG - p_G,$$

if he buys from firm G (p_G is the price that firm G charges),

$$U_B = aB - p_B,$$

if he buys from firm B (p_B is the price that firm B charges), and 0 if he does not buy at all. The taste parameter a is uniformly distributed in the population on the interval $[0, 1]$ - that is, consumers with a low a do not care much about the environment while consumers with a high a care a lot about it. The two firms compete by simultaneously setting prices.

(a) In a figure, illustrate U_G and U_B as a function of a. Using the figure, argue that, *in equilibrium*, it must be that (i) both firms set strictly positive prices, (ii) some strictly positive number of customers will buy from firm B, and (iii) some strictly positive number of individuals will buy from neither firm. (8 points)

(b) Derive the market shares of each firm as a function of qualities G and B and the prices p_G and p_B (hint: some consumers will prefer not to buy at all). (4 points)

(c) Find the Nash equilibrium prices. (6 points)

Next two parts are voluntary - will not be graded, but I will provide a solution

(d) Compute firms' equilibrium profits.

(e) Is it obvious that firm B is better off if it can raise the quality of its product?

Solution

(a) The set of potential customer are generally split into three groups:

1. Those who buy from firm G
2. Those who buy from firm B

3. Those who choose not to buy at all.

Let's first ignore the third possibility and focus on how the various custormers choose among firm G and firm B. It obvious that the customers' choices among the two firms can be characterized as a cutoff: if there is some type $\overline{a}$ who is indifferent between G and B, then any customer with $a > \overline{a}$ will strictly prefer G while any type $a < \overline{a}$ will strictly prefer B. If there is a type $\overline{a}$ that is indifferent between the two firms, given their qualities and prices, then that type will satisfy

$$\overline{a}G - p_G = \overline{a}B - p_B.$$

Hence if $\overline{a}$ is interior (meaning that there will be some individuals who will prefer G and some who prefer B), then, solving,

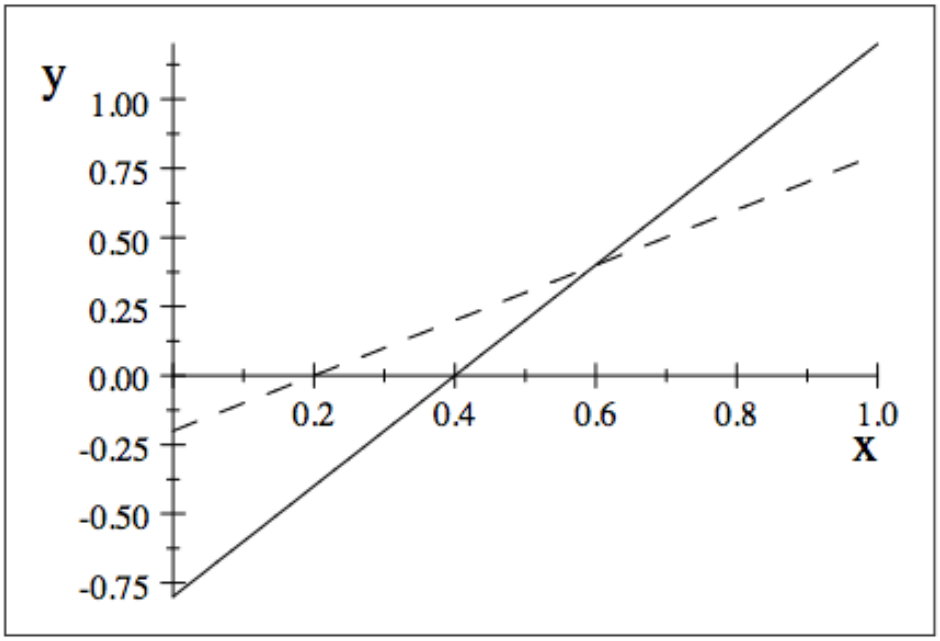
$$\overline{a} = \frac{p_G - p_B}{(G - B)}$$

Note that the only relevant prices are such that $p_G \geq p_B$ (since, by offering a lower quality, firm B can only attract customers by offering a lower price). Note also that the denominator is strictly positive. Hence, for any relevant prices $\overline{a} \geq 0$. However, it could be that $\overline{a} > 1$, implying that no one would choose firm G. This happens when

$$p_G - p_B > G - B$$

i.e. when, intuitively, the price gap is too large relative to the quality gap.

The following figure shows U_G (solid line) and U_B (dashed line) as a function of a for $G = 2$, $B = 1$, $p_G = 0.8$, $p_B = 0.2$. With these numbers, $\overline{a} = 0.6$.



The figure illustrates the choices made by the various individuals depending on their preferences (i.e. their value of a). Individuals with a high value of a choose to by from firm G: for types $a \geq \overline{a}$, $U_G \geq U_B$

and $U_G \geq 0$. There is then a set of individuals who buy from firm B; these are the individuals with $a \geq 0.2$ and $a \leq \overline{a}$. For these individual $U_B \geq U_G$ and $U_B \geq 0$. However, the figure also illustrates that there is a third set of individual: those with $a < 0.2$. For these individuals $0 > U_B > U_G$.

Consider therefor the possibility that there will be a group of individuals who do not buy at all. The figure illustrates that this group will be composed of individuals with low values of a. Indeed, consider the individual with the lowest value of a, i.e. $a = 0$. This person does not value the good at all and would hence never buy at any positive price. However, prices cannot be negative in equilibrium (since that would obviously generate negative profits).

We can now collect some insight. Note that no firm will ever set a price that is below its marginal cost since that could only ever generate negative profits; hence $p_G \geq c > 0$ and $p_B \geq 0$. Moreover, since $p_G > 0$ there will be some consumers who will prefer not buying at all to buying from from G. (These are the individuals with $aG - p_G < 0$ or, equivalently, $a < p_G/G$, and equal to 0.4 in the above example). Finally, in equilibrium, firm B *will* capture some positive number of consumers, set a strictly positive price, and make strictly positive profits. To see this, note that for any values of $G, B > 0$ and $p_G > 0$, firm B can set *some* price $p_B > 0$ that allows them to capture some strictly positive number of customers and therefore generate positive profits. (Formally, suppose that p_B is strictly positive but very small, $p_B = \varepsilon > 0$. In the limit as $\varepsilon \to 0$ firm B will capture all consumers between $a = 0$ and $a = p_G/G$). However, in equilibrium, there *will* also be some consumers who do not buy from either firm. This follows from the fact that, in equilibrium, both firms will set strictly positive prices, $0 < p_B < p_G$.

In equilibrium, it must therefore be that the set of individuals is partitioned into three groups

1. Those with $a < \underline{a}$ will not buy at all, where

$$\underline{a}B - p_B = 0 \Leftrightarrow \underline{a} = \frac{p_B}{B}$$

   (equal to 0.2 in the example above).

2. Those with values of a satisfying $\underline{a} \leq a \leq \overline{a}$ who will buy from firm B

3. Those with values of a such that $a > \overline{a}$, who will buy from firm G.

From the discussion above, we know that, in equilibrium, it has to be that $\underline{a} > 0$ (since, in equilibrium, there will be some individuals who choose not to buy from either firm). Moreover, it has to be that $\underline{a} < \overline{a}$ since, in equilibrium, firm B will necessarily capture some positive number of customers. The only remaining issue is then that it could potentially be that $\overline{a} \geq 1$, implying that, in equilibrium, firm G face zero demand. (We will return to this issue.)

(b) Consider now the market shares. First, firm G will capture all individuals with $a > \overline{a}$. Hence we have that

$$D_G\left(p_G, p_B\right) = 1 - \overline{a} = 1 - \frac{p_G - p_B}{(G - B)}$$

(which is increasing in G, decreasing in B, decreasing in p_G and increasing in p_B). Next, firm B will capture all individuals with $a \geq \underline{a}$ and $a \leq \overline{a}$. Hence

$$D_B\left(p_G,p_B\right) = \overline{a} - \underline{a} = \frac{p_G - p_B}{(G-B)} - \frac{p_B}{B}$$

(which is increasing p_G, decreasing in p_B, decreasing in G and increasing in B).

We maintain the assumption that, in equilibrium, we will have $\overline{a} < 1$ so that $D_G > 0$.

(c) Consider now the profits obtained by each firm. Profits for firm G are

$$\pi_G = \left(p_G - c\right) D_G\left(p_G,p_B\right) = \left(p_G - c\right)\left(1 - \frac{p_G - p_B}{(G-B)}\right)$$

In a Nash equilibrium, the firm set p_G so as to maximize π_G given p_B (and given G and B). The first order condition for profit maximization is

$$\left(1 - \frac{p_G - p_B}{(G-B)}\right) - \left(p_G - c\right)\frac{1}{(G-B)} = 0 \Leftrightarrow$$

$$(G-B) - p_G + p_B = p_G - c$$

Hence, solving yields the best-response function,

$$p_G\left(p_B\right) = \frac{c + (G-B) + p_B}{2}$$

Similarly, for firm B we have that

$$\pi_B = p_B D_B\left(p_G,p_B\right) = p_B\left(\frac{p_G - p_B}{G-B} - \frac{p_B}{B}\right)$$

The first order condition for profit maximization is

$$\left(\frac{p_G - p_B}{G-B} - \frac{p_B}{B}\right) - p_B\left(\frac{1}{G-B} + \frac{1}{B}\right) = 0$$

or

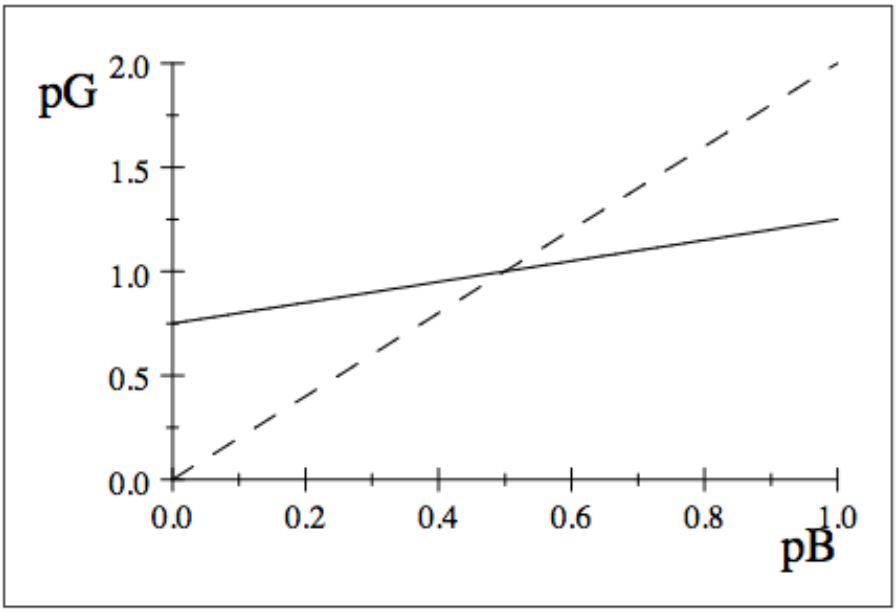
$$\frac{p_G - p_B}{G-B} - \frac{p_B}{B} = \frac{p_B}{G-B} + \frac{p_B}{B}$$

$$\frac{p_G}{G-B} = 2p_B\left(\frac{1}{G-B} + \frac{1}{B}\right) = 2p_B\frac{G}{(G-B)\,B}$$

and hence, solving for firm B's reaction function

$$p_B\left(p_G\right) = \frac{p_G}{2}\frac{B}{G}$$

It is easy to see that the Nash equilibrium is unique. (The following figure illustrates for the case where $c = 1/2$, $G = 2$, $B = 1$, where the solid line is $p_G\left(p_B\right)$ and the dashed line is $p_B\left(p_G\right)$)



Solving for the equilibrium prices is easily done; using firm B's reaction to substitute for p_B in firm G's best-response function, we obtain

$p_G^* = \frac{2G}{4G-B}(c+G-B)$, $p_B^* = \frac{B}{4G-B}(c+G-B)$

Problem 2: Smaller questions - 3 marks for each correct answer

Correct answers: ('b','c','d','f','c','d','b','a','d')

1) Consider the Hotelling mode of horizontal differentiation in the [0,1] interval with two firms. Which statement about the marginal consumer is always true? (graphs might be useful to think about this one)

a) it is the person who is exactly in the middle between the two firms

b) it is the person for whom utility (net of the price and the transportation cost) of buying from firm A is the same as buying from firm B

c) it is the person who enjoys consuming the goods of the two companies equally

d) it is the person located at 0.5

2) Patents provide better incentives to innovate in a market where

a) some firm already has a monopoly

b) there is an oligopoly of two firms

c) there is perfect competition

3) Consider a market with two firms, producing goods of different quality. Is it certain that an equilibrium exists where one firm burns a pile of money in front of consumers to convey useful information about its product? (tip: what kind of game would that be?)

a) yes, if the cost of money for the high quality firm is lower than for the other

b) yes, if the cost of money for the high quality firm is higher than for the other

c) no

d) not enough information to answer this question

4) A cartel may be easier to maintain in a market where:

a) there are many firms

b) the prices and sales quantities of each firm are known only to the firm itself

c) demand for the products is stable

d) the firms sell homogeneous products

e) a and b

f) c and d

5) Cartels have a tendency to self-destruct because:

a) they reduce consumer surplus

b) they set price above marginal cost

c) collusion is not a best response in the one shot market game (i.e. if market interaction is not repeated)

d) each cartel member earns economic profit

6) Which of the following conditions must be true for a firm to price discriminate?

a) the firm must have market power

b) the good cannot easily be resold

c) firm must be able to identify differences in willingness to pay

d) all of the above

7) Consider a model of spatial competition, such as Hotelling's linear city or Salop's competition on a circle, with the firms having exogenous locations.

Reductions in the "transportation" cost, or a consumer's disutility from buying a product that is located further away from her preferred location, increase firms' profits in equilibrium?

a) True

b) False

c) Uncertain

8) At many municipal golf courses, local residents pay a lower fee to play than other golfers do. One necessary condition for the golf course to be able to successfully price discriminate according to residency is that

a) they can check the identification cards of golfers.

b) local resident golfers and other golfers have the same price elasticity of demand to play at the municipal course.

c) there are many golf courses nearby from which golfers can choose.

d) they require all golfers to rent a cart.

9) A firm might use quantity price discrimination if:

a) some consumers tend to buy more units of the product than other consumers

b) consumers are generally willing to pay more for the first few units of the product than for additional units of the product.

c) there is a distinct and observable group with a higher willingness to pay for the product

d) a and b

Problem 3: Medium length question - 12 marks (6 for each question)

Correct answers: ('c','e')

Two companies are thinking of investing in R&D to develop a new space rocket in order to win the X prize, which is worth 240 million pounds. They are only interested in the X prize and will get no other revenue. If they both develop the rocket they share the prize.

If firm Firm A invests, it has to pay 40 million and gets a chance of 1/4 to develop a rocket that would actually qualify for the prize. If firm Firm B invests, it has to pay 60 million and gets a chance of 1/3 to actually develop a qualifying rocket.

i) Which firm(s) invest in R&D, in equilibrium?

a) Firm A

b) Firm B

c) both

d) none

ii) Suppose you now actually own both firms and are risk neutral. Which one will you want to be investing in R&D?

a) firm A

b) firm B

c) both

d) none

e) indifferent between any options from {a, b, c}

f) indifferent between any options from {a, b, c, d}