1 Problem set 6

1)

Consider the modified prisoner's dilemma game and the suggested SPNE (play C in periods 1,...,T-1 and P in period T if the play is (C,C) up to that point, otherwise play (D,D) for the rest of the game)

a) show that it does not pay to deviate in period T

b) show that it does not pay to deviate in period T-1

c) show that it does not pay to deviate in any period t < T-1

d) find other SPNE's (there are a lot!)

e) replace the 7 for (D,C) with 10. Find the minimum number of repetitions such that there is a SPNE where (C,C) is played in the first period.

a) of course it doesn't, P,P is a Nash equilibrium (i.e. there is no profitable deviation)

b) payoff in equilibrium $\pi = 5 + 3$

if I deviate the other player plays D in period T, so $\pi' = 7 + 0$

since $\pi > \pi'$ it does not pay to deviate

c) payoff in equilibrium $\pi = 5 + 5 + 3$

if I deviate the other player plays D in all subsequent periods, so $\pi' = 7 + 0 + 0$

since $\pi > \pi'$ it does not pay to deviate

same is true for any period $t < T - 1$

d) for example, if $T = 5$ consider following SPNE

start with (C,D) in first period and play (C,C) in the second until the last period. In last period play (P,P)

if there is a deviation play D for ever

This is a SPNE since noone wants to deviate.

e) minimum $T = 3$

because if $T = 1$ then it's just a one shot game

if $T = 2$ the punishment from paying (D,D) instead of (P,P) is not big enough to deter a player from defecting in the previous period

2) Consider Bertrand competition with demand D = 20-p, constant marginal cost of 0 and two firms. Let prices be constrained to integers.

a) Show that the stage game has two Nash-equilibria.

b) Now consider the game to be repeated T times. Show that there is a SPNE where both firms set the monopoly price $p^m = 10$ up to $t = T - 3$.

a) There are two equilibria,

$p = 0$ and $p = 1$

At any of these prices it does not pay to deviate.

b) for example set $p_t = 10$ for $t \leq T - 3$

$p_{T-2} = 4$

$p_{T-1} = 2$

$p_T = 1$
If anyone deviates then $p_t = 0$ for all t

3) Osborne 429.1
the condition is
$x + \delta x + \delta^2 x + ... \geq y + \delta + \delta^2 + ...$
$x(1 + \delta + \delta^2 + ...) \geq y - 1 + (1 + \delta + \delta^2 + ...)$
$x\frac{1}{1-\delta} \geq y - 1 + \frac{1}{1-\delta}$
$x \geq (y-1)(1-\delta) + 1$
$\frac{x-1}{y-1} \geq 1 - \delta \Rightarrow \frac{y-x}{y-1} \leq \delta$