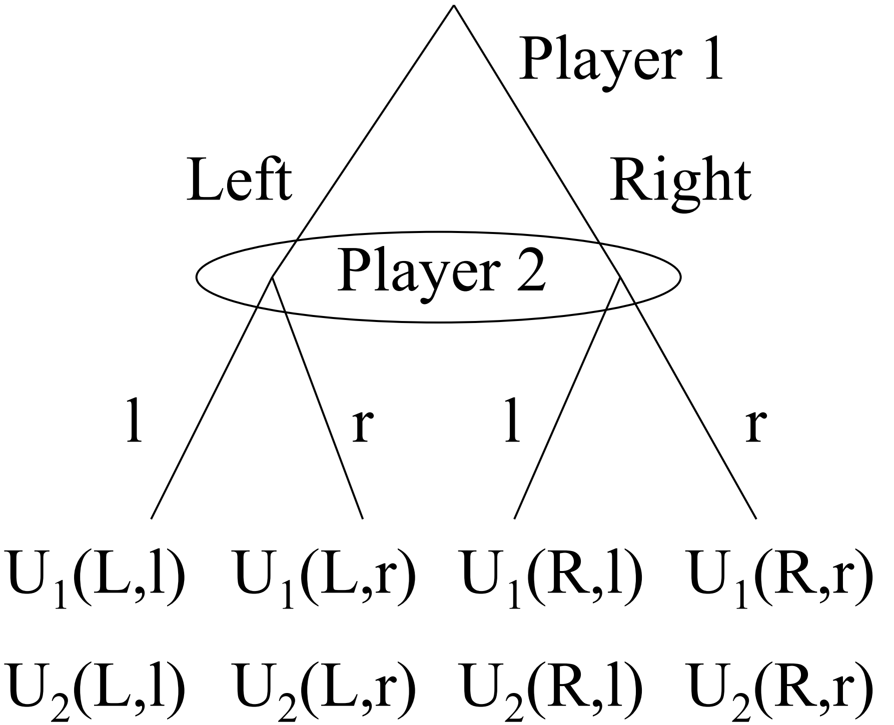
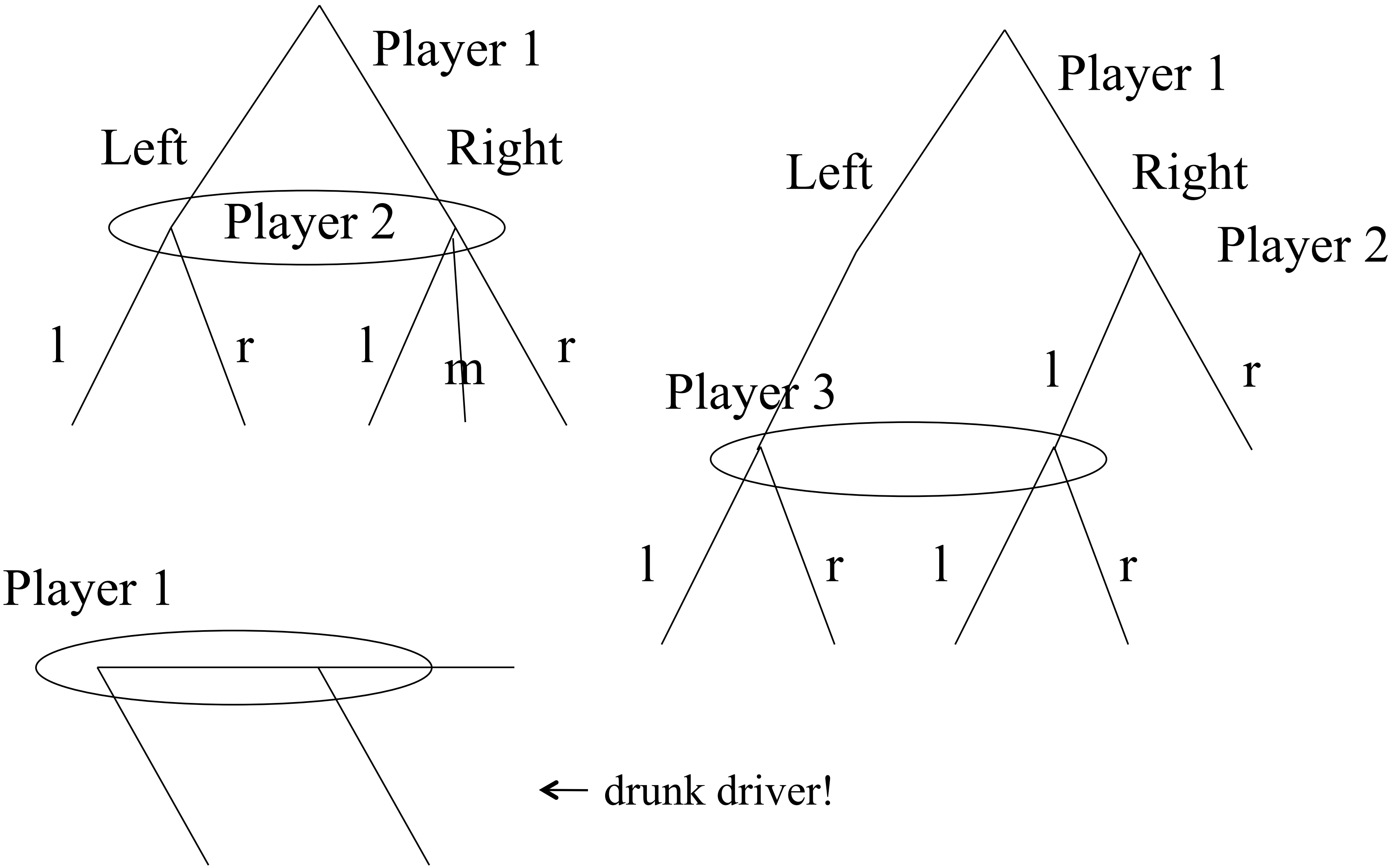
EC3324 Autumn, Lecture #08
Extensive Games with Imperfect Information

- Reading
  - Osborne, Chapter 10
- Learning outcomes
  - describe an extensive form game with imperfect information
  - be familiar with the concept of a signalling game
  - understand the basic idea of a (weak) Perfect Bayesian Equilibrium

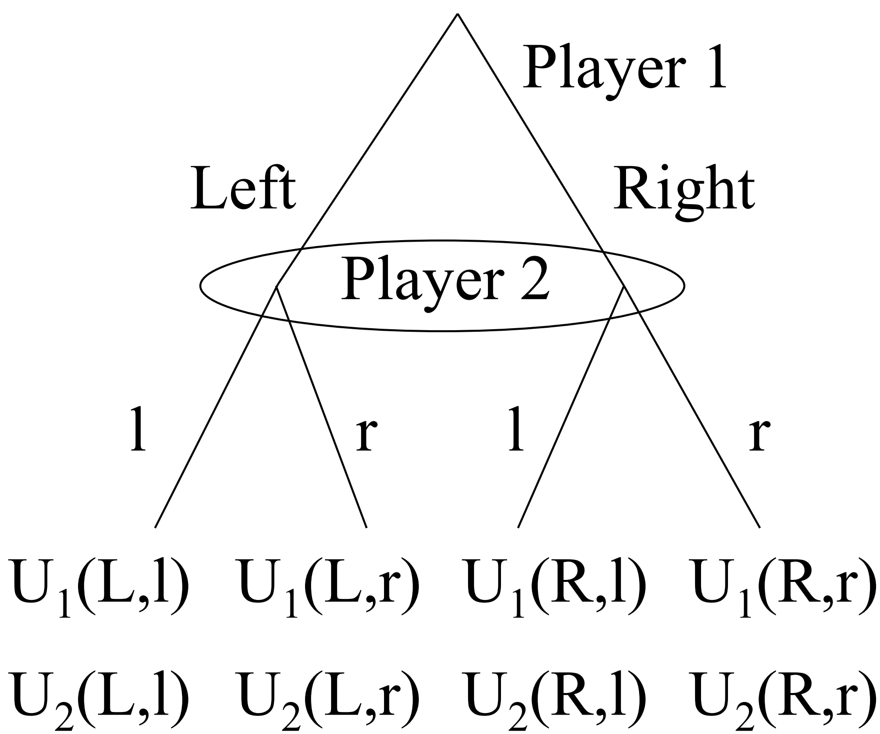
- If a player in an extensive game does not know the history by the time he chooses, the game is one of **imperfect information**
- we can mark what a player knows at the time he moves with an **information set** (consists of indistinguishable histories)
- Player 2 knows that he is in the information set, but not in which specific node
- Hence Player 2's **strategy** cannot condition on Player 1's choice, but a strategy has to prescribe a move for each **information set**



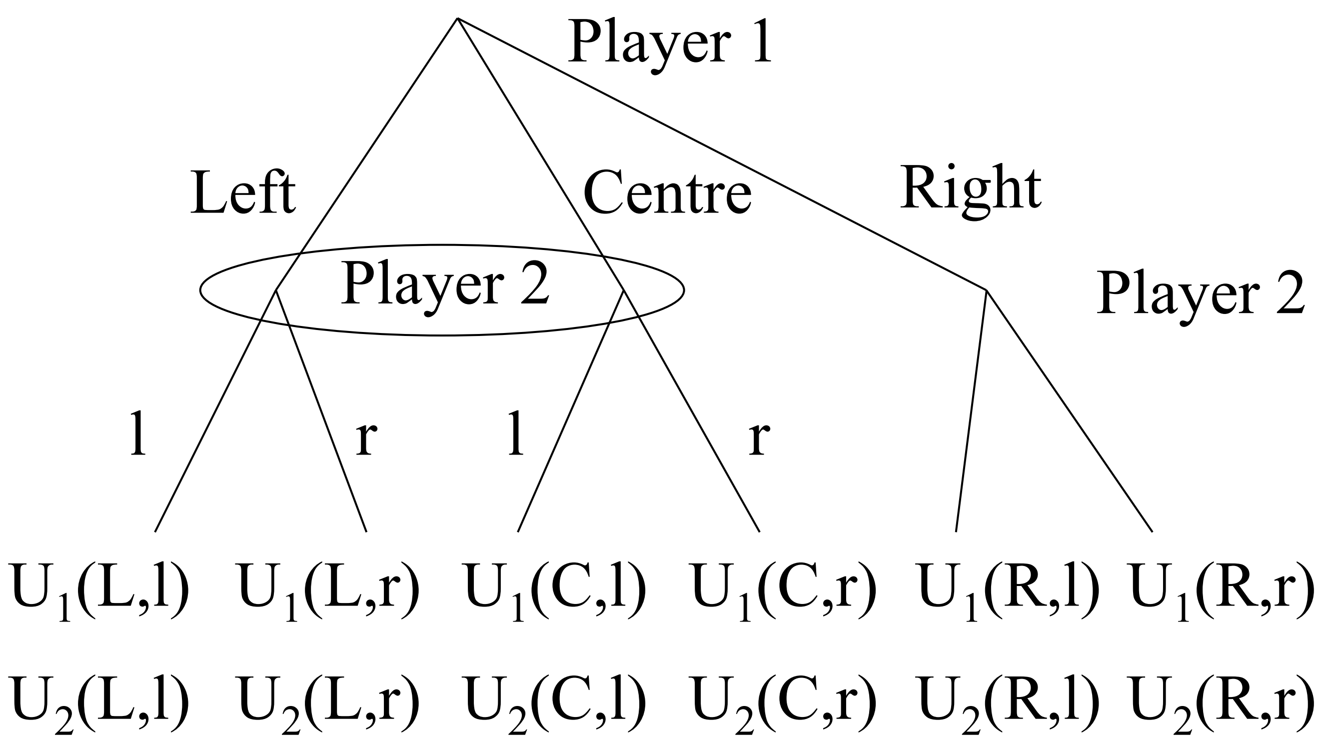
Note: What can an info set NOT look like



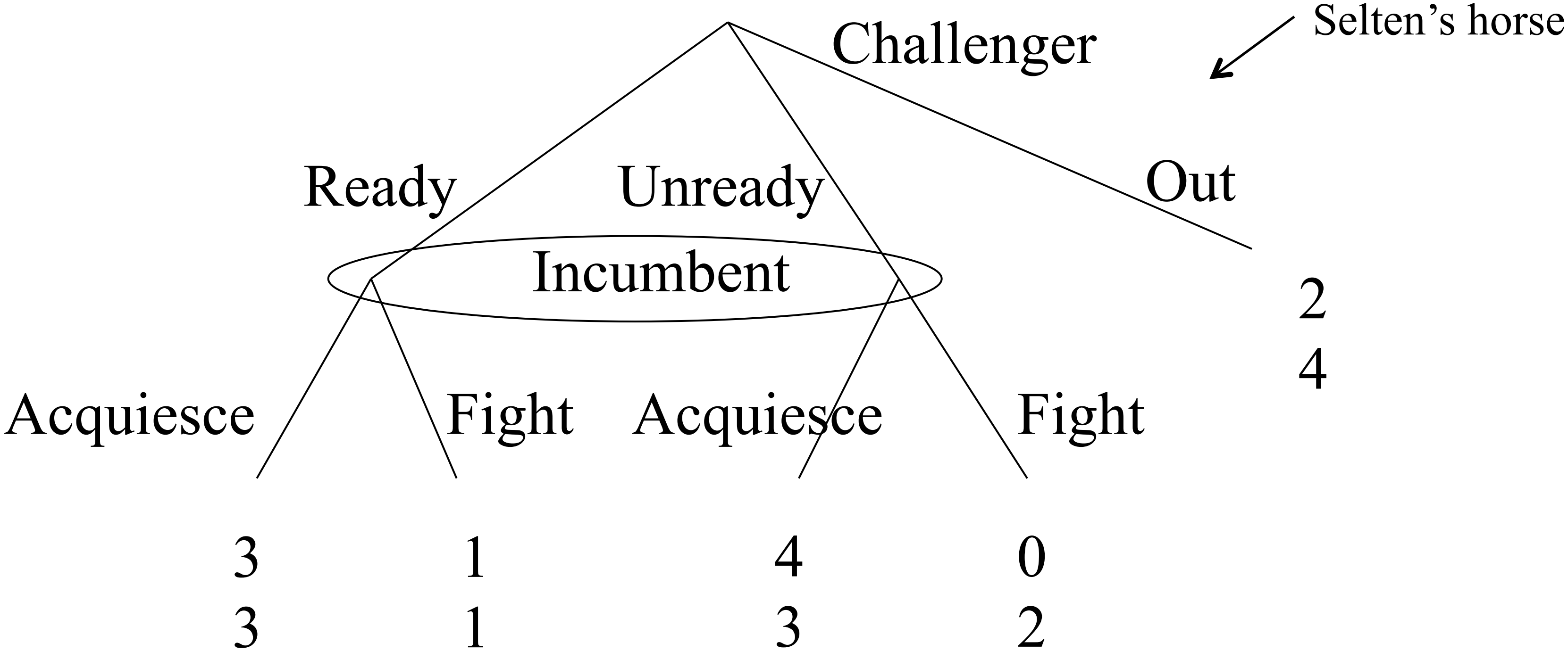
- Games like the one below can be treated like a simultaneous move game
- Player 2 chooses with knowledge as if players choose simultaneously
- We can then consider Nash-equilibria as for simultaneous move games



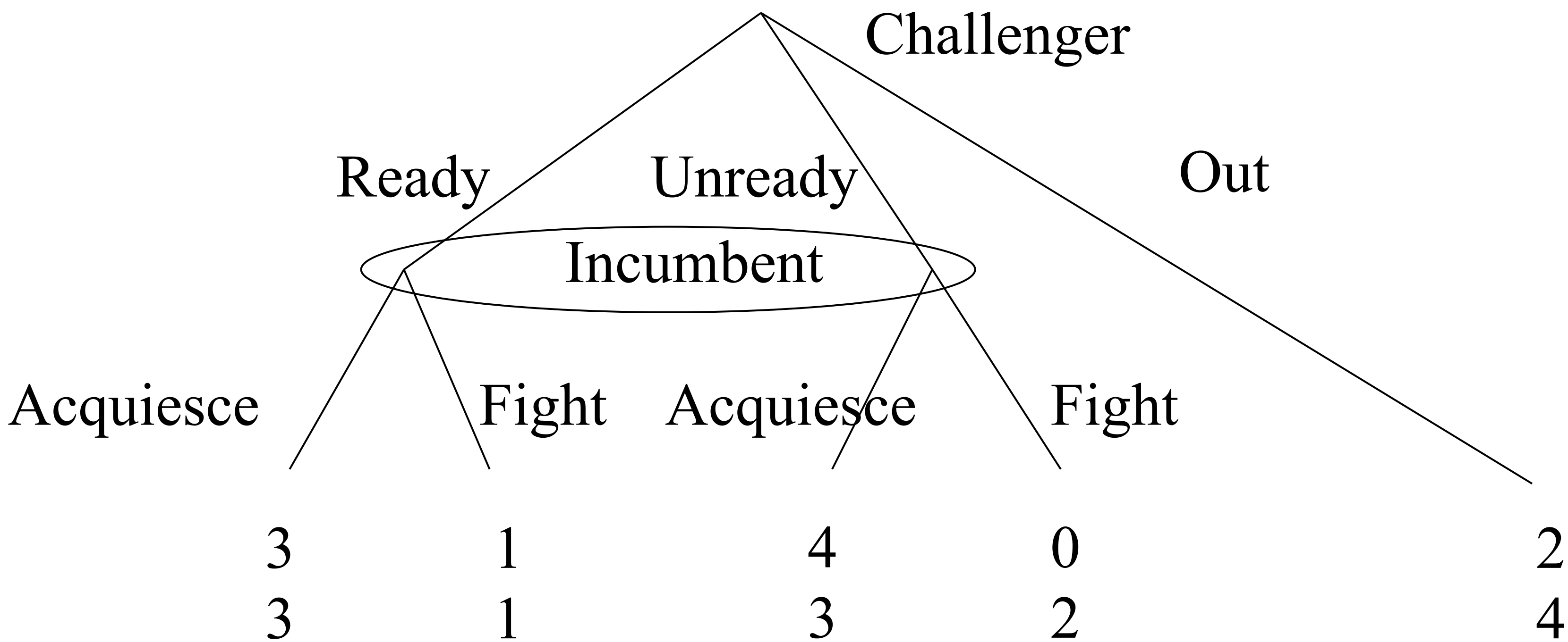
- But the situation differs if Player 2 knows **something** about Player 1's move, but not always everything
- Here Player 2 knows whether Player 1 has chosen right or not, so can condition on **some** knowledge, but does not always know the history



- **Example:** Market Entry Game
- Challenger can enter prepared for fight or unprepared
- Incumbent only knows whether Challenger enters or not, but not whether he is prepared for fight
- **Nash equilibria?**
- (*Unready, Acquiesce*) and (*Out, Fight*)



- (*Out, Fight*) is not convincing:
- Incumbent prefers *Acquiesce* both after *Ready* and *Unready*
- Considering subgame perfect equilibrium does not help:
- The only subgame is the game itself
  - Intuitively, if you cut a subgame at its starting node it should "fall" from the tree, not hang by an info set
- So both (*Unready, Acquiesce*) and (*Out, Fight*) are subgame perfect equilibria
- Want to generalize idea of **sequential rationality** to such games



Generalizing sequential rationality

Definition: Belief system: assigns to each information set a probability distribution over the histories in that information set

Definition: weak Perfect Bayesian equilibrium:

- combination of strategies and beliefs that satisfy:
  - **sequential rationality**: each player's strategy is optimal whenever she has to move given belief and other's strategies
  - **consistency of beliefs**: each player's beliefs are consistent with the strategy profile

weak Perfect Bayesian Equilibrium

- consistency of beliefs requires in particular:

  – in any history that is reached with positive probability given the strategy profile, belief is formed following **Bayes' rule**:

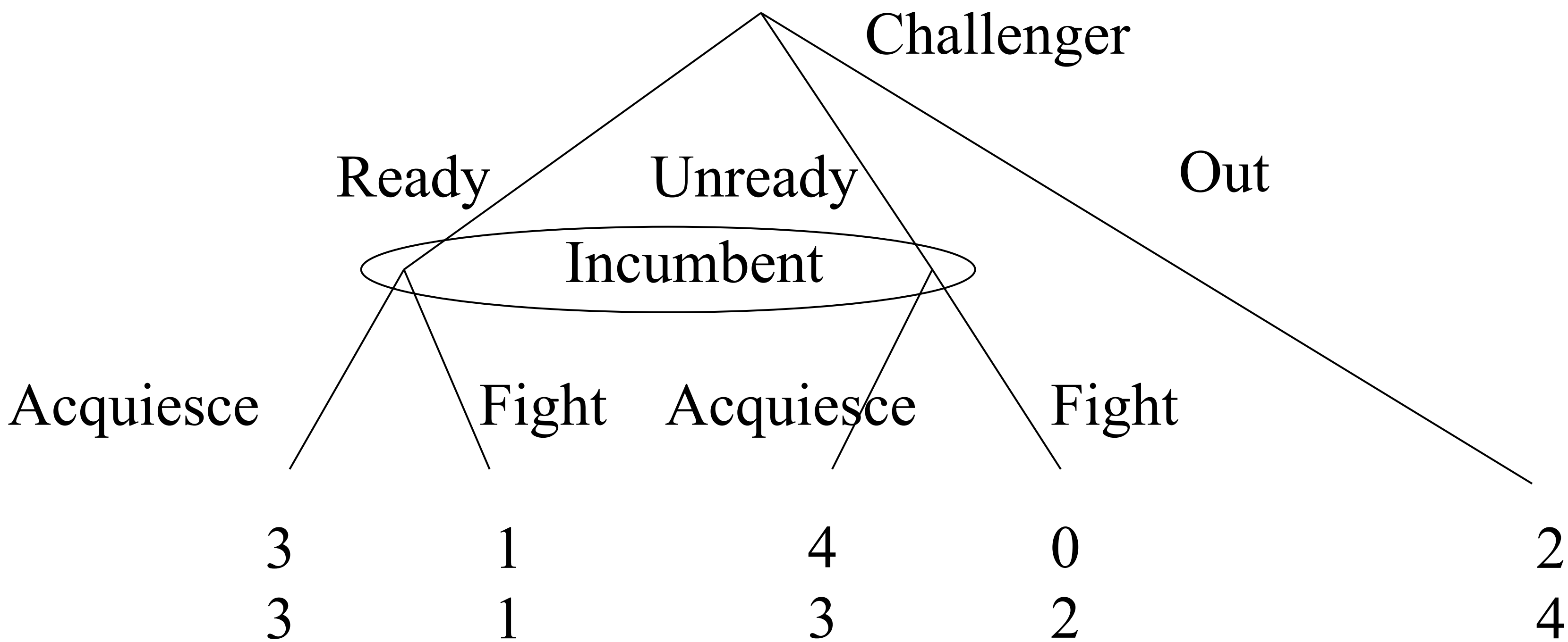
  – for any strategy profile B and history h^* in information set I, the belief is given by

  – $\Pr_B(h^* | I) = \Pr_B(h^*) / \Pr_B(I) = \Pr_B(h^*) / \sum_{h \text{ in } I} \Pr_B(h)$

  – For information sets that are reached with **probability 0**, beliefs are **arbitrary**

- **Perfect Bayesian equilibrium** adds further requirements that assure subgame perfection (e.g. by requiring that explicitly)

- Now consider again the market entry game
- For **any belief** *Acquiesce* is optimal
- Hence in any wPBE Incumbent has to choose *Acquiesce*
- So the only wPBE is (*Unready, Acquiesce*)



Signalling Games

- Apply the idea of a **Bayesian** game to extensive form games:

- **Nature** chooses the **type** of (at least) one player, say Player 1

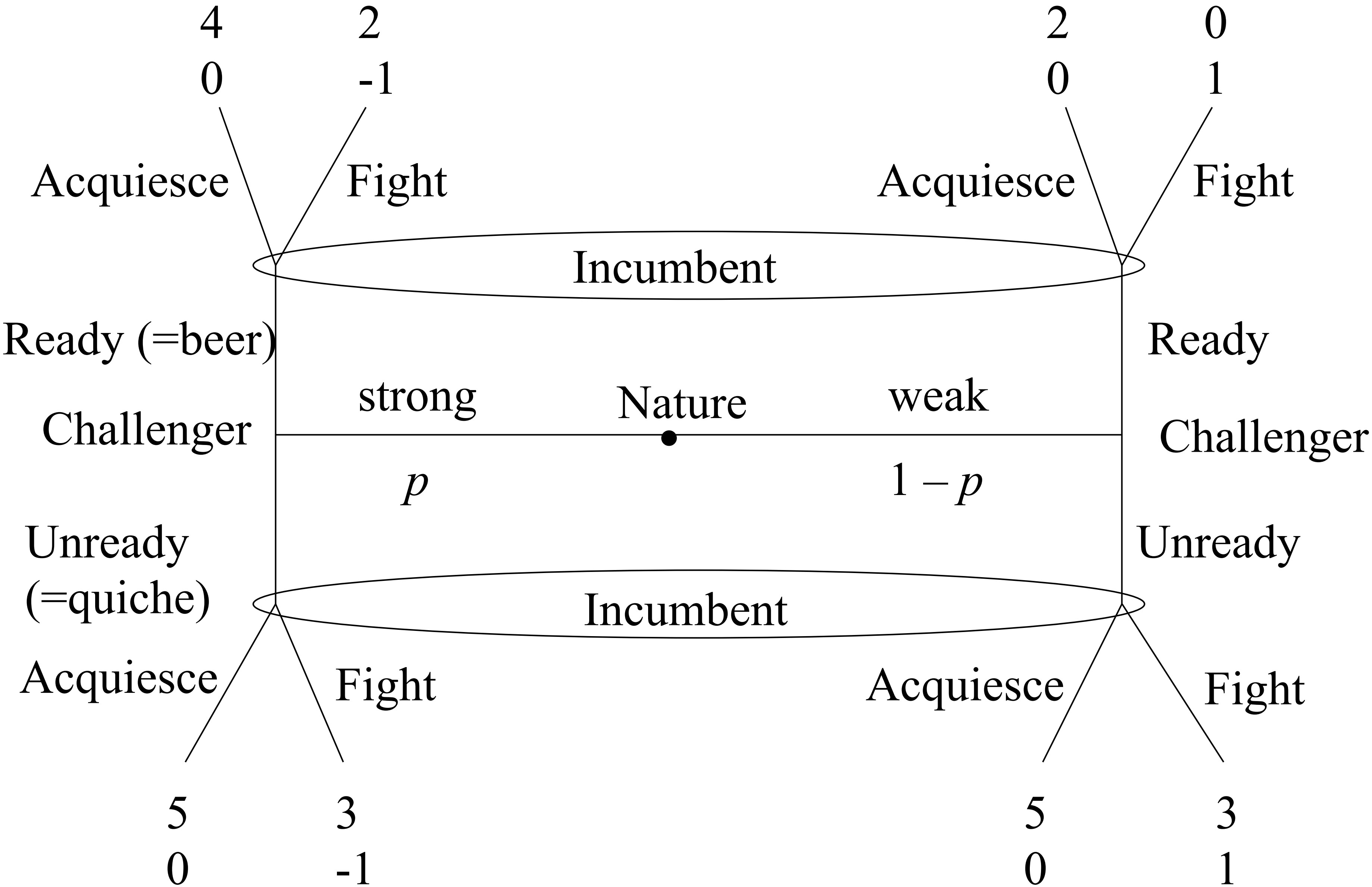
- Player 1 observes his own type

- Player 2, before choosing, does not observe the **type** of Player 1 but does observe the **actions** of Player 1

- Put differently, Player 2 does not observe Nature's action, so this is an extensive game with imperfect information

Signalling Games

- This implies the opportunity for **signalling:** Player 1 can reveal his type through an action
  - often, one type will want to reveal his type, the other wants to hide it and will want to **imitate** the other type
  - if imitating is cheap, we have **pooling equilibria:** all types choose the same action
  - if imitating is too costly, there can be **separating equilibria**: different types choose different actions and the action reveals the type



Entry as signaling game (challenger's payoff on top)

- Note first that the weak challenger prefers *unready* whatever action the incumbent takes, so in any wPBE the weak challenger must choose *unready*
- That leaves two possibilities for equilibria:
  1. Strong challenger chooses *ready* (separating equilibrium)
  2. Strong challenger chooses *unready* (pooling equilibrium)
- Consider 1: Strong challenger chooses ready
  - both information sets of incumbent are reached
    - beliefs are hence given by Bayes' rule
    - thus Pr(*strong* | *ready*) = Pr(*weak* | *unready*) = 1
  - thus incumbent chooses *acquiesce* after *ready* and *fight* after *unready*
  - no type has an incentive to deviate
- so this is a (separating) wPBE (for any *p*)

- Consider 2: Strong challenger chooses *unready*
  - only bottom information set (*unready*) of incumbent is reached
    - beliefs here are given by Bayes' rule
      - thus Pr(*strong* | *unready*) = *p*; Pr(*weak* | *unready*) = 1 – *p*
  - $E(A \mid unready) = 0$; $E(F \mid unready) = -p + 1 - p = 1 - 2p$
  - $E(A \mid unready) \geq E(F \mid unready) \Leftrightarrow 0 \geq 1 - 2p \Leftrightarrow p \geq ½$
  - we also need to specify strategy given *unready*, although it's never reached, to check if challenger would want to deviate
    - since probability of *unready* is 0, beliefs are not restricted
    - now, if incumbent chooses *acquiesce* after *unready,* no type of challenger would want to deviate, irrespective of what the incumbent would choose after *ready*
- So if $p \geq 1/2$, there are (pooling) wPBE where both types of challengers choose *unready,* incumbent chooses *acquiesce* after *unready* and something consistent with the (arbitrary) belief after *ready*

- Now consider: Strong challenger chooses *unready*, but $p \leq 1/2$
  - then *fight* is best response of incumbent after *unready*
  - if incumbent chose *acquiesce* after *ready,* then strong challenger would prefer to deviate
  - so incumbent must choose *fight* after *ready* (at least with probability $\geq 1/2$)
  - this requires for incumbent's belief: Pr(*strong* | *ready*) $\leq 1/2$
  - *ready* occurs with probability 0, so beliefs are not restricted
  - So if $p \leq 1/2$, there are (pooling) wPBE where both types of challengers choose *unready,* incumbent chooses *fight* after both *unready* and *ready* and believes the challenger to be strong with probability $\leq 1/2$ after both *unready* and *ready*
- This class of equilibria does not survive **refinements** such as the **intuitive criterion**
  - the argument is that the weak challenger would never want to deviate to *ready*, undermining the belief Pr(*strong* | *ready*) $\leq 1/2$)

Problem set #08

Suggested problems (with solutions on the book's website)

1. Osborne Ex 316.1
2. Osborne Ex 318.2
3. Osborne Ex 331.2