Lecture 5

Today's agenda

- A Duopoly Version of the Cournot Model
- A Linear Example with n Firms
- Problems (with solution):
  - Merger in a Cournot competition
  - A Comparison of a Differentiated Bertrand Duopoly and a Differentiated Cournot Duopoly

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Aims

- Be able to characterize the Cournot equilibrium.
- Understand the comparative welfare properties of Cournot and Bertrand outcomes.

Tirole, Ch. 5 (including the introduction to Part II), pp. 205-226 (except 5.7.1.3).

A Duopoly Version of the Cournot Model I

- Two firms produce identical products.
  - As the products are identical, inverse demand is a function of the firms' total output:

$$p = P(q_1 + q_2),$$

    where q_1 is Firm 1's output and q_2 is Firm 2's output.
  - As usual, we assume that the demand function is downward-sloping: $P'(q_1 + q_2) < 0$.
- No other producers are able to enter the market.
- The firms' cost functions are denoted $C_1(q_1)$ and $C_2(q_2)$, respectively.
  - We assume that each firm's cost function is strictly increasing and convex: $C_i'(q_i) > 0$ and $C_i''(q_i) \geq 0$.

A Duopoly Version of the Cournot Model II

- *The firms choose their outputs* (as opposed to the Bertrand model, in which they choose their prices).
  - The interpretation: Once the firms have chosen the outputs, some non-modelled "auctioneer" is picking a price that ensures that market demand equals the firms' aggregate output.
  - Each firm's strategy set is the set of all non-negative real numbers: $A_i = \Re_+$ (where A_i is the generic notation for a strategy set used in the previous lecture and in Tirole).
- The firms interact just once and they make their output decisions, q_1 and q_2, simultaneously.
- The firms' profit functions (their payoffs) are therefore

$$\Pi^1(q_1, q_2) = q_1 P(q_1 + q_2) - C_1(q_1)$$

  and

$$\Pi^2(q_1, q_2) = q_2 P(q_1 + q_2) - C_2(q_2).$$

A Duopoly Version of the Cournot Model III

- Further assumptions:
  - Each firm's profit function is concave in the own quantity: $\Pi_{11}^{1}(q_1, q_2) < 0$ or

$$\underbrace{2P'(q_1+q_2)}_{<0}\underbrace{-C_1''(q_1)}_{\leq 0}+q_1\underbrace{P''(q_1+q_2)}_{?}<0$$

    (and similarly for Firm 2). This holds if the demand function is not too convex (the last term).
  - The firms' choice variables are strategic substitutes: $\Pi_{12}^{1}(q_1, q_2) < 0$ or

$$\underbrace{P'(q_1+q_2)}_{<0}+q_1\underbrace{P''(q_1+q_2)}_{?}<0$$

    (and similarly for Firm 2). This also holds if the demand function is not too convex (the last term).

A Duopoly Version of the Cournot Model IV

- Note that, in principle, the first inequality could hold but not the second, in which case the profit function is concave but the choice variables are strategic complements ($\Pi_{12}^{i}(q_1, q_2) > 0$).

A Duopoly Version of the Cournot Model V

A Nash equilibrium of this Cournot model

- The pair of output quantities (q_1^*, q_2^*) is a **Cournot-Nash equilibrium** if neither firm can increase its profits by unilaterally choosing some other quantity, *given the equilibrium quantity of its rival*:

$$\Pi^1 (q_1^*, q_2^*) \geq \Pi^1 (q_1, q_2^*) \qquad \text{for all } q_1 \in \Re_+$$

and

$$\Pi^2 (q_1^*, q_2^*) \geq \Pi^2 (q_1^*, q_2) \qquad \text{for all } q_2 \in \Re_+.$$

- This is often called a Cournot (or Cournot-Nash) equilibrium.
    - But think of it as a Nash equilibrium of the Cournot model.
    - Exactly as before with the Bertrand model: we stick to *a single equilibrium concept* but vary the rules of the game.

A Duopoly Version of the Cournot Model VI

- In an equilibrium in which $q_1^* > 0$ and $q_2^* > 0$, each firm's first-order condition must hold.
    - Therefore, we can characterize the equilibrium by the following two equations:

$$\begin{aligned} & \Pi_1^1 (q_1^*, q_2^*) \\ = \; & P (q_1^* + q_2^*) - C_1' (q_1^*) + q_1^* P' (q_1^* + q_2^*) = 0, \end{aligned} \tag{1}$$

$$\begin{aligned} & \Pi_2^2 (q_1^*, q_2^*) \\ = \; & P (q_1^* + q_2^*) - C_2' (q_2^*) + q_2^* P' (q_1^* + q_2^*) = 0. \end{aligned}$$

    - Interpretation:
        1. The two first terms of equation (1), price less MC, represent the addition to profits from the *last produced unit*.
        2. The third term represents the negative effect of selling this extra unit on the *infra-marginal units* (due to a lower price).

A Duopoly Version of the Cournot Model VII

- Under the perfect competition, effect 2 vanishes (as firms take price as give).
- For a monopoly, effect 2 is stronger (as the firms output equals industry output).
- Implication: equilibrium price under Cournot duopoly is in between the monopoly price and MC.
  - More generally: the more firms there are in the Cournot market, the closer price is to MC.
- We can also characterize the equilibrium by using best-reply functions.
  - Let's draw the graphs of the two firms' best-replies in a diagram with q_2 on the vertical axis and q_1 on the horizontal axis.
  - Since these variables are strategic substitutes ($\Pi^i_{12}(q_1, q_2) < 0$), we know that both best replies are downward-sloping.
  - Moreover, Firm 2's best reply is flatter than Firm 1's, due to a commonly made *stability assumption*.
  - The equilibrium is at the crossing of the two best replies.

A Duopoly Version of the Cournot Model VIII

- Using the diagram, we can also do a comparative statics exercise:
  - Suppose Firm 1's marginal cost increases.
  - Then Firm 1's best reply shifts left.
  - The new equilibrium (the crossing point) moves north-west.
  - That is, $q_1^* \searrow$ and $q_2^* \nearrow$.
- Exercise: Do the same experiment under the assumption that the best replies are upward-sloping! Does this change the result? Why?

A Linear Example with n Firms I

- Consider the following model:
  - n identical firms.
  - Indirect demand is given by

$$p = a - b \sum_{i=1}^{n} q_i.$$

  - All firms have the same constant marginal cost c (with $a > c \geq 0$), and there are no fixed costs.
  - Therefore,

$$\begin{aligned} \Pi^i &= q_i \left(a - b \sum_{j=1}^{n} q_j \right) - c q_i \\ &= q_i \left(a - c - b \sum_{j=1}^{n} q_j \right). \end{aligned}$$

A Linear Example with n Firms II

- Solving the model:
  - Firm i solves (taking all others' output as given):

$$\max_{q_i \geq 0} q_i \left(a - c - b \sum_{j=1}^{n} q_j \right).$$

  - The FOC:

$$\left(a - c - b \sum_{j=1}^{n} q_j \right) - b q_i = 0. \tag{2}$$

  - Note that there are n FOCs like the one above — one for each firm.
  - We can easily prove that all firms must choose the same output in equilibrium:

A Linear Example with n Firms III

- Adding all the FOCs in (2) yields

$$n\left(a-c-b\sum_{j=1}^{n}q_j\right)-b\sum_{j=1}^{n}q_j=0.$$

- Solve for $b\sum_{j=1}^{n}q_j$:

$$b\sum_{j=1}^{n}q_j=\frac{n(a-c)}{n+1}.$$

- Plug this expression for $b\sum_{j=1}^{n}q_j$ back into (2) and solve for q_i:

$$bq_i=a-c-\frac{n(a-c)}{n+1}$$

or

$$q_i=\frac{a-c}{b(n+1)}\equiv q^*.$$

- We have found that all firms produce the same amount, and we have also found an expression for this amount (q^*).

A Linear Example with n Firms IV

- We can also calculate:

$$Q^*\equiv nq^*=\frac{n(a-c)}{b(n+1)},$$

$$p^*-c\equiv a-bQ^*-c=\frac{a-c}{n+1},$$

$$\Pi^*\equiv(p^*-c)\,q^*=\frac{(a-c)^2}{b(n+1)^2},$$

- *Exercise:* What happens with these expressions as n grows and approaches infinity? What is the interpretation of this?

Problem 1 I

Consider a market with three firms (i = 1, 2, 3), which have identical marginal costs $c_1 = c_2 = c_3 = 0$. The inverse demand function is given by $p = 1 - Q$, where $Q = q_1 + q_2 + q_3$.

(a) Compute the Nash-Cournot-equilibrium.

(b) Assume that two of the three firms merge. Show that the profit of the merging firms decreases.

(c) What happens if all three firms merge?

Problem 2 with solutions I

A comparison of a differentiated Bertrand duopoly and a differentiated Cournot duopoly

- Consider a market with two firms. The indirect demand functions for the firms' goods are

$$p_1 = \alpha - q_1 - \gamma q_2,$$

$$p_2 = \alpha - q_2 - \gamma q_1,$$

where α and γ are parameters satisfying $\alpha > 0$ and $-1 < \gamma < 1$. The firms have the same cost function, which is given by

$$C(q_i) = cq_i,$$

where c is a parameter satisfying $0 \leq c < \alpha$.

(a) Calculate the Cournot-Nash equilibrium. What is the market price for each good in this equilibrium? Are q_1 and q_2 strategic substitutes or strategic complements?

Problem 2 with solutions II

(b) Invert the two indirect demand functions so that you get two direct demand functions.

(c) Calculate the Bertrand-Nash equilibrium. What is the market price for each good in this equilibrium? Are p_1 and p_2 strategic substitutes or strategic complements?

(d) Which model (quantity setting or price setting) gives rise to the lowest market price?

Problem 2 with solutions III

Solutions

(a) *Calculate the Cournot-Nash equilibrium. What is the market price for each good in this equilibrium? Are q_1 and q_2 strategic substitutes or strategic complements?*

- Firm 1's profit:

$$\pi_1 = (\alpha - c - q_1 - \gamma q_2)\, q_1.$$

FOC:

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_1} &= -q_1 + (\alpha - c - q_1 - \gamma q_2) = 0 \\ &\Leftrightarrow \quad q_1 = R_1(q_2) = \frac{\alpha - c - \gamma q_2}{2}, \end{aligned}$$

where $R_1(q_2)$ is firm 1's best-response function. Clearly, this is downward-sloping for positive values of γ, and it is upward-sloping for negative values of γ.

Problem 2 with solutions IV

- For firm 2 we have, by symmetry,

$$R_2(q_1) = \frac{\alpha - c - \gamma q_1}{2}.$$

- Therefore, q_1 and q_2 are strategic substitutes for $\gamma > 0$, and strategic complements for $\gamma < 0$.
- Solving for the equilibrium yields

$$\left(q_1^C, q_2^C\right) = \left(\frac{\alpha - c}{2+\gamma}, \frac{\alpha - c}{2+\gamma}\right).$$

Problem 2 with solutions V

- The market price for good 1 at this equilibrium:

$$\begin{aligned} p_1^C &= \alpha - q_1^* - \gamma q_2^* \\ &= \alpha - \frac{\alpha - c}{2+\gamma} - \gamma \frac{\alpha - c}{2+\gamma} \\ &= \frac{(2+\gamma)\alpha - (1+\gamma)(\alpha - c)}{2+\gamma} \\ &= \frac{\alpha + (1+\gamma)c}{2+\gamma}. \end{aligned}$$

By symmetry,

$$p_2^C = \frac{\alpha + (1+\gamma)c}{2+\gamma}.$$

Problem 2 with solutions VI

(b) *Invert the two indirect demand functions so that you get two direct demand functions.* Inverting yields

$$q_1 = \frac{1}{1-\gamma^2}\left[(1-\gamma)\,\alpha - p_1 + \gamma p_2\right],$$

$$q_2 = \frac{1}{1-\gamma^2}\left[(1-\gamma)\,\alpha - p_2 + \gamma p_1\right].$$

(c) *Calculate the Bertrand-Nash equilibrium. What is the market price for each good in this equilibrium? Are p_1 and p_2 strategic substitutes or strategic complements?*

Problem 2 with solutions VII

- Firm 1's profit:

$$\pi_1 = (p_1 - c)\,\frac{1}{1-\gamma^2}\left[(1-\gamma)\,\alpha - p_1 + \gamma p_2\right].$$

FOC:

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_1} &= \frac{1}{1-\gamma^2}\left[(1-\gamma)\,\alpha - p_1 + \gamma p_2\right] \\ &\quad -\frac{1}{1-\gamma^2}\,(p_1 - c) \\ &= 0 \\ &\Leftrightarrow\; p_1 = R_1(p_2) = \frac{(1-\gamma)\,\alpha + c + \gamma p_2}{2},\end{aligned}$$

where again $R_1(p_2)$ is firm 1's best-response function. Clearly, this is upward-sloping for positive values of γ, and it is downward-sloping for negative values of γ.

Problem 2 with solutions VIII

- For firm 2 we have, by symmetry,

$$R_2(p_1) = \frac{(1-\gamma)\alpha + c + \gamma p_1}{2}.$$

- Therefore, p_1 and p_2 are strategic complements for $\gamma > 0$, and strategic substitutes for $\gamma < 0$. [The exact opposite to what we have above in the Cournot model.]
- Solving for the equilibrium yields

$$\left(p_1^B, p_2^B\right) = \left(\frac{(1-\gamma)\alpha + c}{2-\gamma}, \frac{(1-\gamma)\alpha + c}{2-\gamma}\right).$$

Problem 2 with solutions IX

(d) *Which model (quantity setting or price setting) gives rise to the lowest market price?*

- Setting $p_1^C > p_1^B$ and then simplifying yield

$$\begin{aligned} p_1^C &> p_1^B \Leftrightarrow \frac{\alpha + (1+\gamma)c}{2+\gamma} > \frac{(1-\gamma)\alpha + c}{2-\gamma} \\ &\Leftrightarrow \gamma^2\alpha > \gamma^2 c \Leftrightarrow \alpha > c, \end{aligned}$$

which, by assumption, is always true. Therefore, we always have $p_1^C > p_1^B$: in this model, Bertrand competition always gives rise to a lower market price.