Candidate Name………………. Student Id No……………

ECONOMICS DEPT

NON - ASSESSED TEST

For Internal Students of
Royal Holloway

COURSE UNIT : EC3324

TITLE: Topics in Game Theory

Date of Test

Time Allowed: 1 hour

Instructions to candidates:

ANSWER *ONE* OF THE QUESTIONS

DO NOT TURN OVER UNTIL TOLD TO BEGIN

Question 1: Penalty shooting re-mixed

Consider the following situation. A kicker and a goalkeeper face each other in a penalty. The kicker and the gaolie decide simultaneously in which corner to shoot or jump (we see everything from the perspective of the kicker, so if both choose "right", they are in the same corner). The kicker may miss the goal altogether. Otherwise, if the kicker does not miss the goal and the goalkeeper is in the other corner, the kicker scores for sure. If the kicker does not miss the goal and the goalie is in the same corner, the goalie catches the ball with positive probability, but not with certainty.

If the kicker misses the goal, he assigns this event a value of -2 and the goalie assigns it a value of 1. If the kicker scores, he assigns this a value of 1 and the goalie assigns this a value of -1. If the goalie catches the ball, he assigns this a value of 2 and the kicker assigns this a value of -1. Both players evaluate a combination of actions by the expected value for this combination, so for the kicker U = Pr(miss) (-2) + Pr(catch) (-1) + Pr(score) 1 with the appropriate probabilities for the given combination of actions.

The kicker is better in shooting to the right side. When shooting to the right, he misses the goal with probability 0.2, while if he shoots to the left he misses with probability 0.4 (the kicker is English and the game is important, so these probabilities are realistic). On either side, whenever the goalie jumps in the correct corner, conditional on the kicker not missing the goal, the goalie is equally likely to catch the ball as he is to let it pass. Remember that when the kicker does not miss the goal and the goalie is in the wrong corner, the kicker scores for sure.

a) Calculate the payoffs (as expected values as explained above) for each combination of actions and fill them into this table

		goalie	
		L	R
kicker	L	-0.8 , 0.7	-0.2 , 0.2
	R	0.4 , -0.6	-0.4 , 0.6

(5 marks)

b) Is there a pure-strategy equilibrium? Give a brief explanation.

(2 marks)

Nope, there is no cell in the matrix that is a mutual best response.

c) Find the mixed-strategy equilibrium of the above game.

(6 marks)

Let p the probability of L for the kicker and q for the goalie. In equilibrium $q=1/7$ and $p=4/7$

d) Assume the kicker practises a bit and manages to reduce the probability of missing the goal when he shoots to the right to $p=0.05$ (everything else stays the same). Find the Nash-equilibrium.

(4 marks)

RR

e) Assume instead that practise leads to a reduction of the probability to miss the goal when he shoots to the left to p=0.20, (the probability to miss when shooting to the right stays at p=0.20). Find the equilibrium by a simple argument, without any computation (if you cannot come up with a simple argument, then you have to do the computation)

(4 marks)

They now mix 0.5 , 0.5

f) Comparing your equilibria in c) and e) shows a surprising result (if you have done the calculations correctly): when getting better shooting to the left, the kicker will actually do this with a lower probability. Try to explain why this happens. **(4 marks)**

The important thing in a mixed strategy equilibrium is that you choose probabilities so the other player is indifferent. If you get better at shooting towards one side you have to do it less often to keep the other player indifferent.

(Total 25 marks)

Question 2

a) In the Table given below, find values b and c such that Player 1 and Player 2 both have a strictly dominated strategy (do not consider domination by mixed strategies at the moment). State which strategy is dominated by which other strategy.

		Player 2			
		W	X	Y	Z
	T	7,8	8,c	1,2	0,5
Player 1	M	5,5	3,3	b,1	3,4
	B	0,2	2,5	4,9	2,8

(4 marks)

b>4 and c<5 is one solution.

b) Solve the game through iterated elimination of dominated strategies given the values of b and c you have chosen in part (a). Clearly state the order in which you eliminate the dominated strategies. State the Nash-equilibrium.

(4 marks)

M dominates B, then Z dominates X and so on. TW is left

c) Given the values for b and c you found in part (b), are there any mixed-strategy equilibria (that are not degenerate, i.e. pure equilibria)? (No need to compute it if your answer is "yes") Explain briefly.

(3 marks)

No.

d) Now find values of b and c such that there are two strict pure Nash equilibria, different from the equilibrium you found in (b). State these equilibria clearly.

(4 marks)

$c>8$ and $b<4$

e) Given the values for b and c you found in part (d), are there any mixed-strategy equilibria (that are not degenerate, i.e. pure equilibria)? Explain if your answer is "no". Compute one such equilibrium if your answer is "yes".

(4 marks)

f) Let b=3. Is any of Player 1's actions strictly dominated by a mixed strategy? If yes, find such a mixed strategy. If not, explain.

(3 marks)

No. All rows have at least one element that is the highest in the column.

g) Let c=7. Is any of Player 2's actions strictly dominated by a mixed strategy? If yes, find such a mixed strategy. If not, explain.

(3 marks)

First three columns have elements that are weakly highest in their row. Fourth cannot be dominated because the weight on start, Y would have to be 8/9 to dominate elemnt 8 but then it is not possible to dominate the payoff of 4 in cell MZ.

(Total 25 marks)

END