Problem set 4

Osborne 106.2

i)

BOS

	Bach	**Stravinsky**
B	2 ,1	0
S	0	1 , 2

when player 1 is indifferent between going to her less preferred concert in the company of player 2 and the lottery in which with probability 1/2 she and player 2 go to different concerts and with probability 1/2 they both go to her more preferred concert, the Bernoulli payoffs that represent her preferences satisfy the condition

$u_1(S,S) = 1/2u_1(S,B) + 1/2u_1(B,B)$

If we choose $u_1(S,B) = 0$ and $u_1(B,B) = 2$, then $u_1(S,S) = 1$. Similarly, for player 2 we can set $u_2(B,S) = 0, u_2(S,S) = 2$, and $u_2(B,B) = 1$. Thus the Bernoulli payoffs in the left panel of Figure 23.1 are consistent with the players' preferences.

Simpler:

	B	**S**
B	x , y	0
S	0	y , x

we need $y = 1/2x + \frac{1}{2}0 \Longrightarrow y = \frac{x}{2}$
e.g. $x = 2, y = 1$

ii)
need $y = 1/4x + \frac{3}{4}0 \Longrightarrow y = \frac{x}{4}$
e.g. $x = 2, y = 1/2$

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1)

	L	**R**
T	6,0	0,6
B	3,2	6,0

Table 1:

$p = prob\{T\}$, $q = prob\{L\}$

p_1 :
$U(T) = 6q$
$U(B) = 3q + 6(1 - q) = 6 - 3q$
to get indifference
$6q = 6 - 3q \Longrightarrow q = 2/3$

p_2 :
$U(L) = 2(1 - p)$
$U(R) = 6p$
to get indifference
$6p = 2 - 2p \Longrightarrow p = 1/4$

equilibrium $(1/4, 2/3)$

2)

	L	**R**
T	0,1	0,2
B	2,2	0,1

Table 2:

note T weakly dominated. If P1 mixes P2 must play R.
R is optimal if
$p + 2(1 - p) < 2p + 1 - p$
$\Rightarrow p + 2 - 2p < p + 1 \Rightarrow 2 - p < p + 1$
$\Rightarrow p > 1/2$

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	no eff	**eff**
no eff	0,0	0,-c
eff	-c,0	1-c,1-c

Table 3:

pure Nash (ne, ne) and (eff, eff)
mixed:
$U(noeffort) = 0$
$U(effort) = (1 - p)(-c) + p(1 - c)$
to get indifference set them equal
$\Rightarrow p = c$

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n citizens, k support A, m support B
$2 \leq k \leq m$

$$U(vote) = \begin{cases} 2-c, \text{ if candidate wins} \\ 1-c, \text{ if tie} \\ -c \text{ else} \end{cases} \qquad U(abstain) = \begin{cases} 2, \text{ if candidate wins} \\ 1, \text{ if tie} \\ 0 \text{ else} \end{cases}$$

Supporters of A vote with probability p
k supporters of B vote with certainty, others abstain

For A's supporters
$U(v) = (1-c)p^{k-1} - c(1-p^{k-1}) = p^{k-1} - c$
$U(novote) = 0$
set them equal
$p = c^{\frac{1}{k-1}}$

B's supporters
For the abstainers we have that either all A's vote and there is a tie, or at least one of them doesn't and B wins. So the expected utility of an abstaining B is

$$p^k + 2(1-p^k)$$

Voting would yield a certain victory and payoff $2-c$. Thus we have that following must hold $p^k + 2(1-p^k) \geq 2-c \Rightarrow c \geq p^k$

For those who vote

$$U(v) = -c + 1p^k + 2(1-p^k) \text{ (1 if there is a tie, 2 if B wins)}$$

For non voters

$$U(nv) = 1kp^{k-1}(1-p) + 2[1-p^k - kp^{k-1}(1-p)]$$
(she gets 1 in case of a tie and 2 in case of no tie and no win of A)

For voting to pay off

$$U(v) > U(nv) \Rightarrow 1 > c$$

The probability of voting p increases in c, this means turnout rises when costs increase! This result would seem to be a bit counterintuitive, but remember we are looking at a mixed strategy equilibrium, where players are playing as if to keep others indifferent.

	L	**M**	**R**
T	2,2	0,3	1,2
B	3,1	1,0	0,2

Table 4:

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L dominated by mixture of M and R

$p = prob\{T\}$

$q = prob\{M\}$

$$B_1(q) = \begin{cases} 0, \text{ if } q > 1/2 \\ [0,1], \text{ if } q = 1/2 \\ 1, \text{ if } q < 1/2 \end{cases}$$

$$B_2(p) = \begin{cases} 0, \text{ if } p > 2/3 \\ [0,1], \text{ if } p = 2/3 \\ 1, \text{ if } p < 2/3 \end{cases}$$

in eq $(p,q) = (2/3, 1/3)$

so the equilibrium of the game is

$(2/3, 1/3), (0, 1/2, 1/2)$

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pure equilibria

(B,L),(T,R)

are there equilibria with T and L&M or L&R or L&M&R?

no

one with T and M&R?

yes, P2 is indifferent if 1 chooses T

for 1 to choose T we need $prob(r) \geq prob(m)$

eq with B and L&M or M&R or L&M&R? No.

B and L&R?

if 1 chooses B 2 is indiff between L&R.

For 1 to choose B we need $2l + r < 3l \Longrightarrow r > l$

so eq (B,(l,o,r)) with $l \geq 1/2$ and $r = 1 - l$

eq with T&B and L&R? No if $t > 0$ there can be no indifference between L&R

eq with T&B and M&R? No if $t < 1$ $U(R) > U(M)$

eq with T&B and L&M&R?

No
$U(L) = U(R)$ requires t=0
$U(M) = U(R)$ requires t=1

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	A	**B**
A	1,1,1	0,0,0
B	0,0,0	0,0,0

Table 5: A

	A	**B**
A	0,0,0	0,0,0
B	0,0,0	4,4,4

Table 6: B

let $x = prob_1\{A\}, y = prob_2\{A\}, z = prob_3\{A\}$
pure Nash (A, A, A) (B, B, B)
$U_1(A) = yz1$
$U_1(B) = (1-y)(1-z)4$
from indifference
$(1-y)(1-z)4 = yz$

due to symmetry $x = y = z$
$(1-y)(1-y)4 = y^2$
$y = 2(1-y)$
$y = 2/3$

Show the following

Proposition 1 *A 2x2 game with two pure strict Nash equilibria always has a mixed strategy equilibrium that is not a pure strategy equilibrium.*

Write down a generic 2x2 game

	L	**R**
T	a,b	c,d
B	e,f	g,h

Table 7:

Since the equilibria are strict they have to be in diagonally opposed corners of the matrix. Suppose w.l.o.g. they are T,L and B,R.

Then $a > e$ and $g > c$
But then there is $q = prob\{L\}$ such that
$qa + (1 - q)c = qe + (1 - q)g$
Similarly there is $p = prob\{T\}$ such that
$pb + (1 - p)f = pd + (1 - p)h$