Game Theory

Problem set 3

59.2

Payoffs

$\pi_i = q_i(P(q_1 + q_2) - c) - f$ if $q_i > 0$

maximising we get

$$\text{BR}_i = \frac{a-c-q_2}{2}, \text{ if profit is non negative}$$

Profits at such a production level are

$$(\frac{a-c-q_2}{2})^2 - f$$

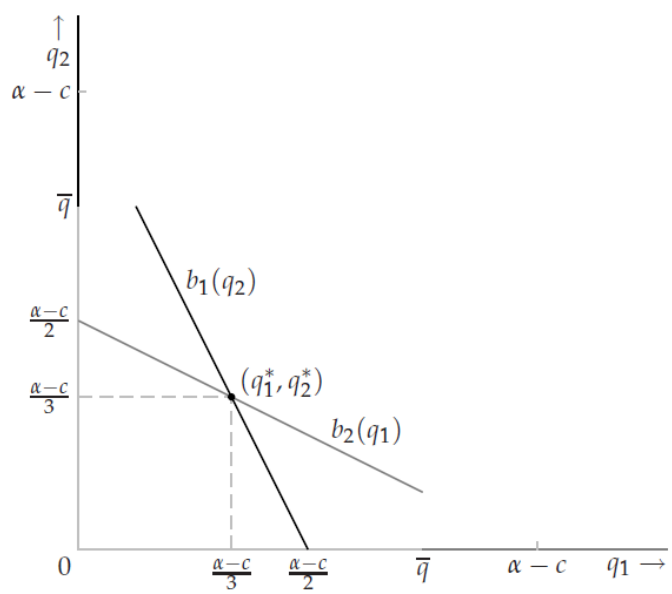
so we need $(\frac{a-c-q_2}{2})^2 > f$ or $q_2 \leq a - c - 2\sqrt{f} = \bar{q}$

then the best response is

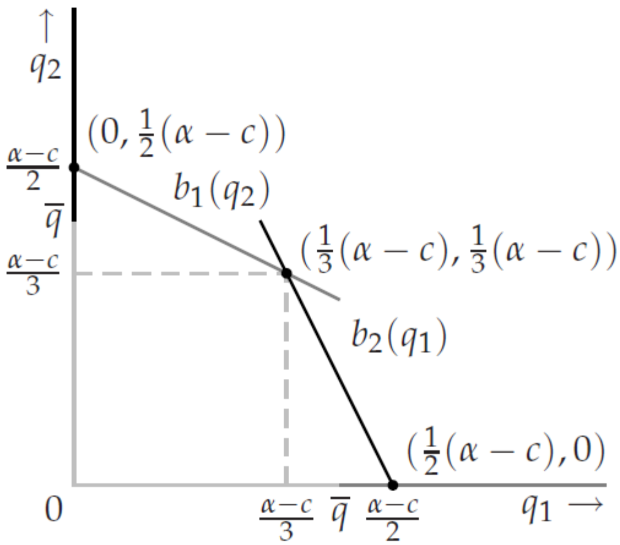
$$BR_1(q_2) = \begin{cases} \frac{a-c-q_2}{2}, \text{if } q_2 < \bar{q} \\ \{0, \frac{a-c-q_2}{2}\}, \text{if } q_2 = \bar{q} \\ 0, \text{if } q_2 > \bar{q} \end{cases}$$

Firms are symmetric, so the same should hold for firm 2.

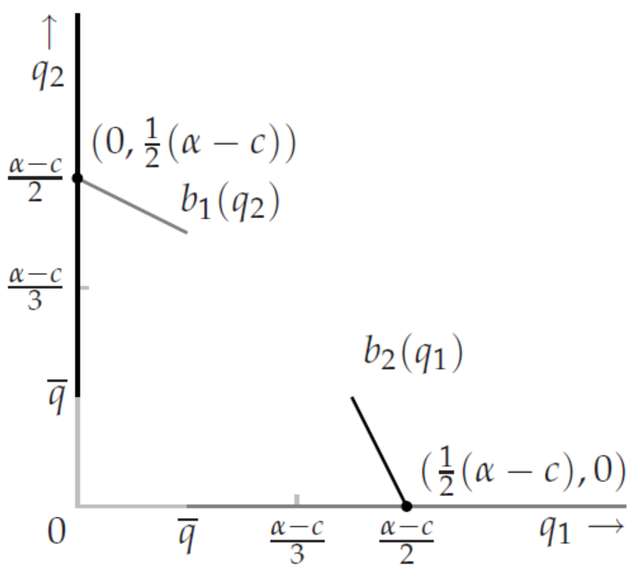
Now, we need to do case distinctions. If f is small enough that $\bar{q} > \frac{a-c}{2} \Rightarrow f < \frac{(a-c)^2}{16}$ then we have following form for the functions



The eq. is then
$(q_1^*, q_2^*) = (\frac{a-c}{3}, \frac{a-c}{3})$
if $\frac{(a-c)^2}{16} < f < \frac{(a-c)^2}{9}$
then



and we have three eq.
If $\frac{(a-c)^2}{9} < f < \frac{(a-c)^2}{4}$



and we have two eq.

If f even larger no firm wants to produce for any quantity of the other firm, so the unique Nash is (0,0)

62.1

if $P(Q^*) <\underline{p}$

the price in the proposed equilibrium is lower than the minimum possible unit cost, so any firm that produces loses money, so $q = 0$ is a profitable deviation

if $P(Q^*+\underline{q}) >\underline{p}$

a firm that is producting 0 or 0+ε (such a firm exists, since there are infinitely many and demand is finite), its profit is zero or 0+δ

If it deviates and produces $\underline{q}$ then the output becomes at most $Q^*+\underline{q}$ so that the price still exceeds $\underline{p}$

(since $P(Q^*+\underline{q}) >\underline{p}$)

So, this is a profitable deviation

69.1

at ($\bar{p}$,$\bar{p}$) profits are 0 as 1 gets the whole market at a profit ($\bar{p}$-c)($\alpha-\bar{p}$) and 2 gets nothing

If any firm raises price its profit remains zero

If either firm lowers price, it receives all demand and loses money

There is no other eq.

If $p_1 = p_2 <\bar{p}$

then 1 loses money, so can deviate by raising price and increase profit

If $p_1 = p_2 >\bar{p}$

then 2 makes zero, can obtain positive by lowering price by ε

If $p_i < p_j$

and i makes pos. profit, then j can raise price a bit over i and still make positive profit instead of zero

If $p_i < p_j$

and i makes zero profit, then i can raise price a bit and make positive profit

If $p_i < p_j$

and i makes negative profit, then i can raise price above j and guarantee zero profit

74.2

In equilibrium both candidates choose median voter position in the largest state (m_1), and there is a tie.

If anyone deviates to more than m_1 she loses in both

If anyone deviates to less than m_1,towards m_2, she gains in 2 but loses in 1 so she loses overall.

There is no other Nash eq. If one or both candidates is away from m_1 there is always a profitable deviation for the losing player: move to m_1 and tie or win.

80.2

If $y_i < y_j$ then j can increase payoff by reducting y by ε, so it must be $y_i = y_j$

Now, if $y < 1$ then any player can deviate to $y+\varepsilon$ and gain the whole output.

So the eq. is (1,1) and payoffs are zero.