Candidate Name………………..   Student Id No………………

ECONOMICS DEPT

NON - ASSESSED TEST

For Internal Students of
Royal Holloway

COURSE UNIT : EC3324

TITLE: **Topics in Game Theory**

Date of Test   **10 November 2010**

Time Allowed: 1   hour

Instructions to candidates:

ANSWER _ONE_ OF THE QUESTIONS

DO NOT TURN OVER UNTIL TOLD TO BEGIN

Question 1: Penalty shooting re-mixed

Consider the following situation. A kicker and a goalkeeper face each other in a penalty. The kicker and the gaolie decide simultaneously in which corner to shoot or jump (we see everything from the perspective of the kicker, so if both choose "right", they are in the same corner). The kicker may miss the goal altogether. Otherwise, if the kicker does not miss the goal and the goalkeeper is in the other corner, the kicker scores for sure. If the kicker does not miss the goal and the goalie is in the same corner, the goalie catches the ball with positive probability, but not with certainty.

If the kicker misses the goal, he assigns this event a value of -2 and the goalie assigns it a value of 1. If the kicker scores, he assigns this a value of 1 and the goalie assigns this a value of -1. If the goalie catches the ball, he assigns this a value of 2 and the kicker assigns this a value of -1. Both players evaluate a combination of actions by the expected value for this combination, so for the kicker $U = Pr(miss)\ (-2) + Pr(catch)\ (-1) + Pr(score)\ 1$ with the appropriate probabilities for the given combination of actions.

The kicker is better in shooting to the right side. When shooting to the right, he misses the goal with probability 0.2, while if he shoots to the left he misses with probability 0.4 (the kicker is English and the game is important, so these probabilities are realistic). On either side, whenever the goalie jumps in the correct corner, conditional on the kicker not missing the goal, the goalie is equally likely to catch the ball as he is to let it pass. Remember that when the kicker does not miss the goal and the goalie is in the wrong corner, the kicker scores for sure.

a) Calculate the payoffs (as expected values as explained above) for each combination of actions and fill them into this table

		goalie	
		L	R
kicker	L		
	R		

(5 marks)

b) Is there a pure-strategy equilibrium? Give a brief explanation.

(2 marks)

c) Find the mixed-strategy equilibrium of the above game.

(6 marks)

d) Assume the kicker practises a bit and manages to reduce the probability of missing the goal when he shoots to the right to p=0.05 (everything else stays the same). Find the Nash-equilibrium.

(4 marks)

e) Assume instead that practise leads to a reduction of the probability to miss the goal when he shoots to the left to p=0.20, (the probability to miss when shooting to the right stays at p=0.20). Find the equilibrium by a simple argument, without any computation (if you cannot come up with a simple argument, then you have to do the computation)

(4 marks)

f) Comparing your equilibria in c) and e) shows a surprising result (if you have done the calculations correctly): when getting better shooting to the left, the kicker will actually do this with a lower probability. Try to explain why this happens. **(4 marks)**

(Total 25 marks)

Question 2

a) In the Table given below, find values b and c such that Player 1 and Player 2 both have a strictly dominated strategy (do not consider domination by mixed strategies at the moment). State which strategy is dominated by which other strategy.

		Player 2			
		W	X	Y	Z
	T	7,8	8,c	1,2	0,5
Player 1	M	5,5	3,3	b,1	3,4
	B	0,2	2,5	4,9	2,8

(4 marks)

b) Solve the game through iterated elimination of dominated strategies given the values of b and c you have chosen in part (a). Clearly state the order in which you eliminate the dominated strategies. State the Nash-equilibrium.
(4 marks)

c) Given the values for b and c you found in part (b), are there any mixed-strategy equilibria (that are not degenerate, i.e. pure equilibria)? (No need to compute it if your answer is "yes") Explain briefly.
(3 marks)

d) Now find values of b and c such that there are two strict pure Nash equilibria, different from the equilibrium you found in (b). State these equilibria clearly.
(4 marks)

e) Given the values for b and c you found in part (d), are there any mixed-strategy equilibria (that are not degenerate, i.e. pure equilibria)? Explain if your answer is "no". Compute one such equilibrium if your answer is "yes".

(4 marks)

f) Let b=3. Is any of Player 1's actions strictly dominated by a mixed strategy? If yes, find such a mixed strategy. If not, explain.

(3 marks)

g) Let c=7. Is any of Player 2's actions strictly dominated by a mixed strategy? If yes, find such a mixed strategy. If not, explain.

(3 marks)

(Total 25 marks)

END