LECTURE 8:
NOISY BEHAVIOUR, QUANTAL RESPONSE EQUILIBRIA AND LEVEL-K

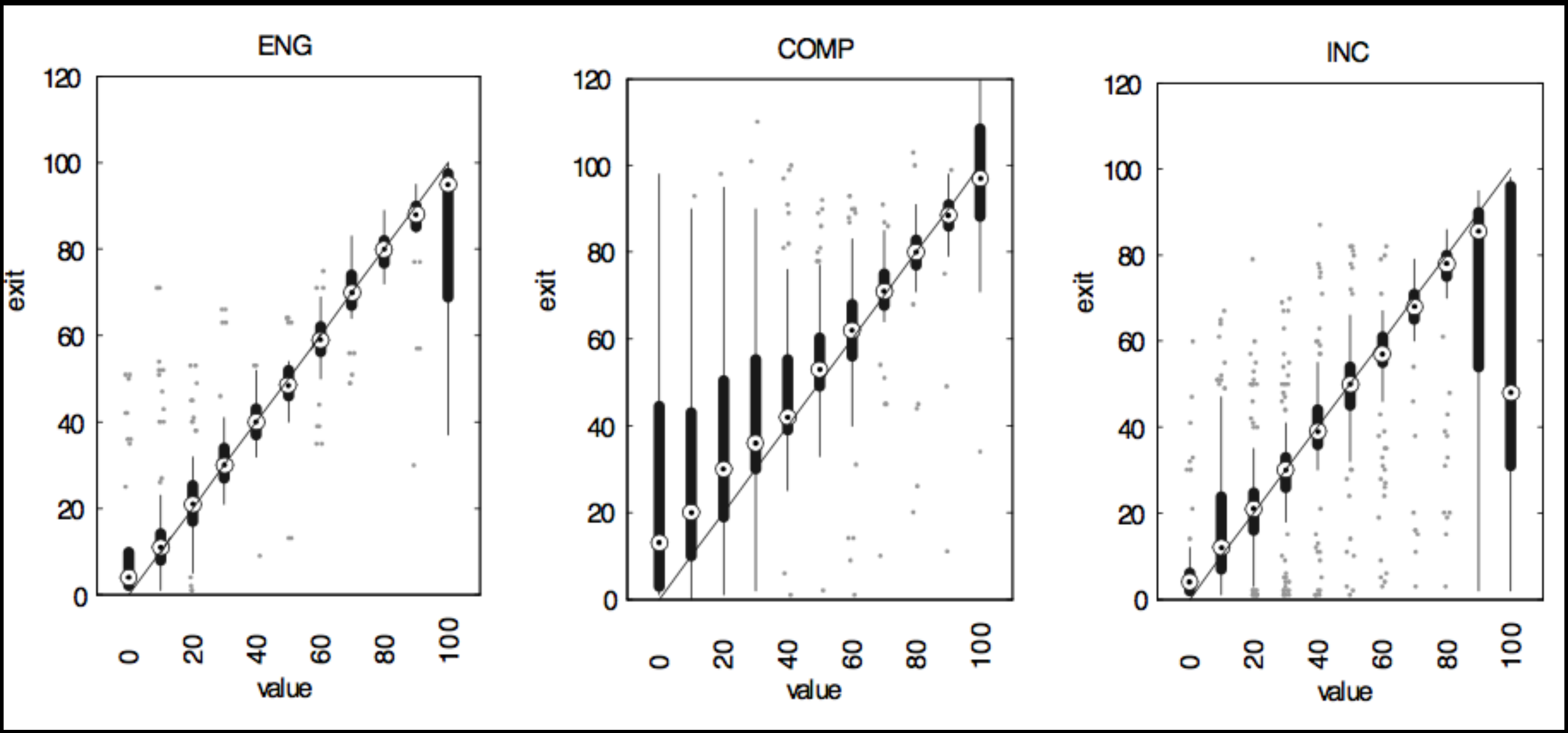
- Reading
  - Goeree & Holt, Ten little treasures, 2001 AER
  - McKelvey & Palfrey 1995 GEB
  - Nagel 1995 AER
- Learning outcomes
  - Understand how expected payoff functions can influence play
  - Be familiar with Quantal Response Equilibria and methods to calculate them

Auction with Resale

Georganas 2011

- Standard English auction for one unit
  - With n bidders
  - IPV in [0,100]
- Twist: there is a second stage where winner can resell the good to an other bidder
  - Chooses a reserve price
  - Other bidders can see and decide whether they are interested
  - If more than 1 interested there is a new English auction

Auctions with Resale: result



Explanation

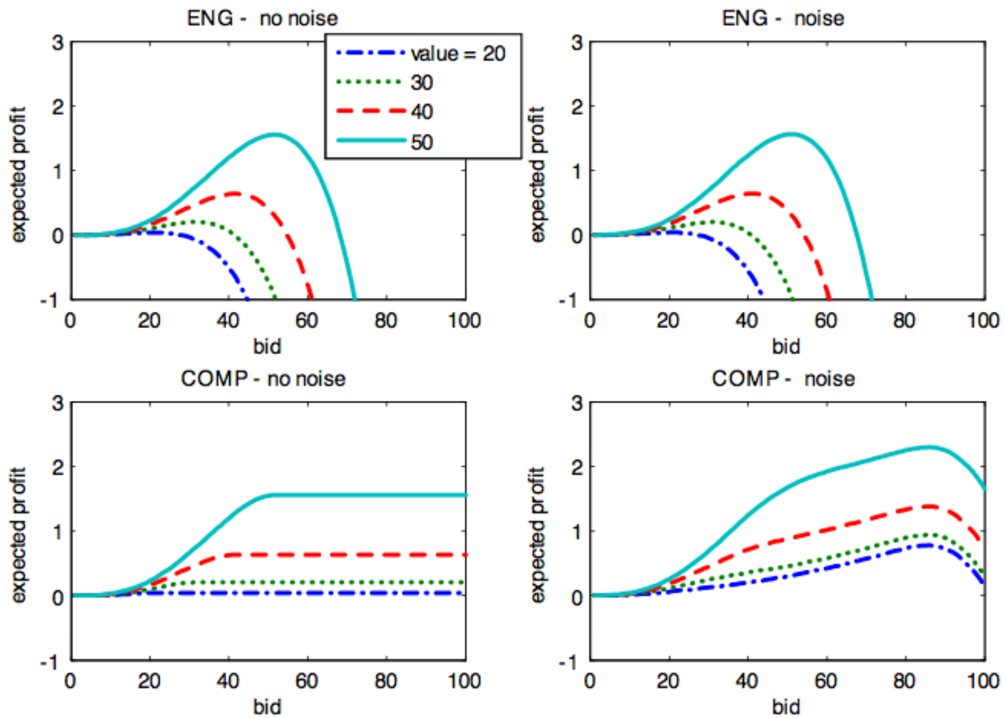


Figure 4: Expected utilites in ENG (upper two figures) and COMP (lower two) without and with noise (normally distriubuted with a σ of 9). The curves are drawn for private use value signals equal to 20, 30, 40 and 50. In the lower left panel utility is very flat but still maximized at a bid equal to value.

Equilibrium plus noise

- Noisy equilibrium is not the same as Nash + mistakes
  - Noise can drive behavior very far away from equilibrium
    - A player's errors can change another's best responses
  - Games of this type
    - Guessing games
    - Auctions with resale
    - Centipede
    - etc

Quantal response equilibrium

McKelvey - Palfrey 1995

- An equilibrium concept that systematically introduces noise
- Nash equilibrium= consistent beliefs + best response
- In QRE players play *better* responses
  - Strategies with higher expected payoff chosen more often
  - But not with probability one
  - Another interpretation: avoiding costly mistakes

The error structure in QRE

- Luce (1959):
  1. response probabilities are an increasing function of the strength of the stimulus f(U)
  2. probabilities of all possible choices have to add up to one

$$p_j=f(U_j)/\Sigma f(U)$$

- For practical purposes we need to assume a specific functional form
  - Simplest: linear $p_j=U_j/\Sigma U$
    - What about negative payoffs?
  - logistic errors $p_j=e^{-\lambda U}/\Sigma e^{-\lambda U}$
- Quantal Response *Equilibrium* is then a fixed point same as Nash
  - I have some beliefs
  - Given beliefs I can calculate my expected payoffs
  - Transform payoffs using logit or linear and play according to that
  - Equilibrium when my beliefs consistent with (noisy) play of others

QRE example in normal form games

	L	R
U	3,3	0,0
D	0,0	1,1

- Row has belief that column plays left with prob λ
- Expected payoff is then
  - $E\pi[U]=3\lambda+(1-\lambda)0$
  - $E\pi[D]=\lambda 0+(1-\lambda)1$
- Quantal Response with Luce errors
  - Prob of playing up $\mu=E\pi[U]/(E\pi[U]+E\pi[D])$
  - $\mu=3\lambda/(2\lambda+1)$
- Do same for Column
  - Game is symmetric, so $\mu=3\mu/(2\mu+1)$
  - $\mu=0$ or $\mu=1$

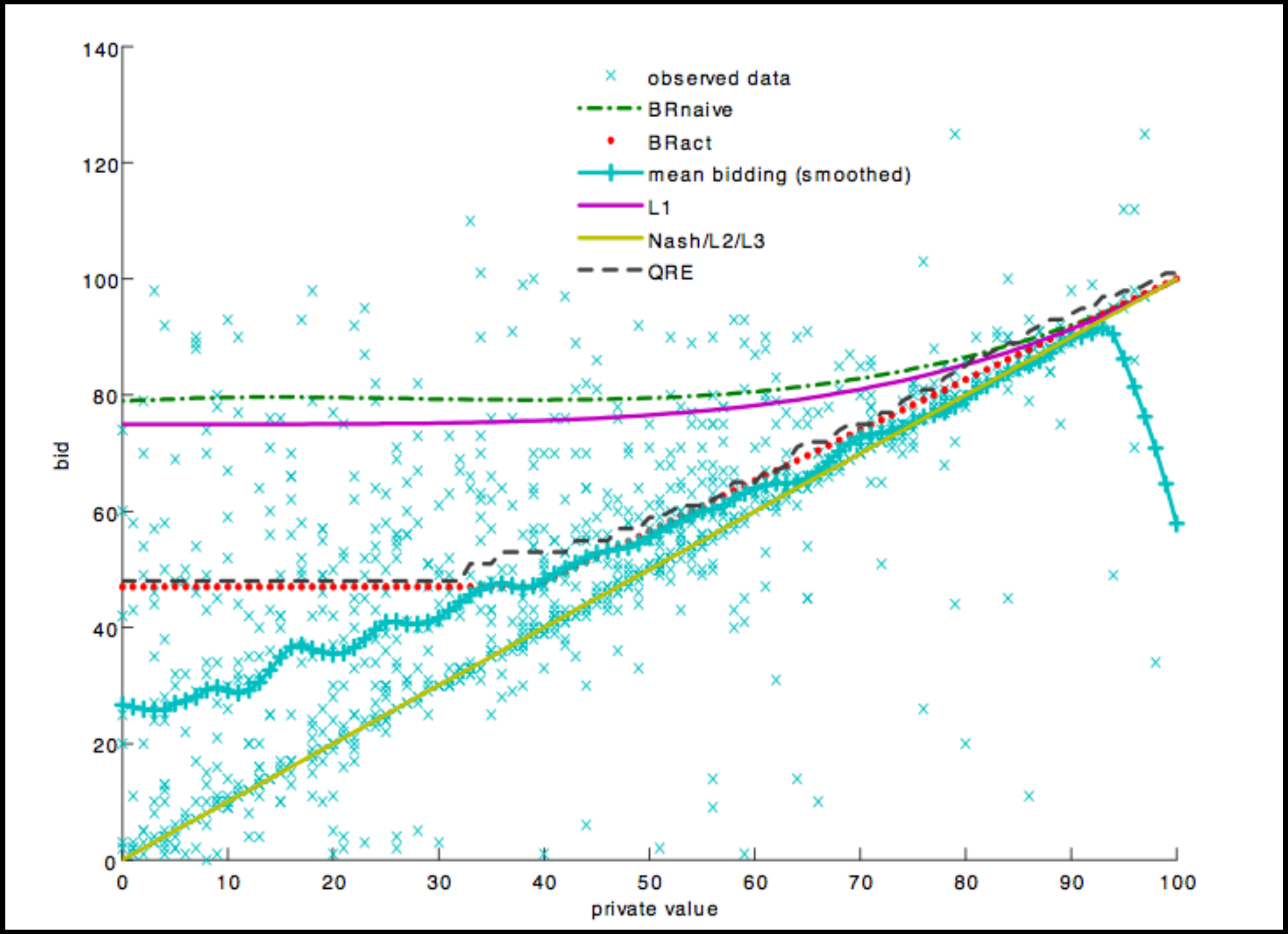
How to calculate QRE

- QRE yields a system of equations
- For large strategy spaces impossible to calculate algebraically
- Numerical methods needed
  1. Tracing method (see *gambit* software)
  2. Iteration method

Fitting QRE to data

- For applications the logit version is the most flexible
  - λ can be interpreted as degree of rationality, as for $\lambda \to \infty$ QRE approached Nash
- The idea is to fit a QRE to the data using maximum likelihood
  - Find the λ that makes choice probabilities most likely
  - E.g. if people play right with probability ½ find λ such that the associated QRE predicts play of right with probability as close to ½ as possible

QRE and Auctions with Resale



QRE and first price auctions
(Goeree, Holt & Palfrey 2002 JET)

Discrete value first price auctions

Low value treatment: Values 0, 2, 4, 6, 8, 11

High value treatment: Values 0, 3, 5, 7, 9, 12

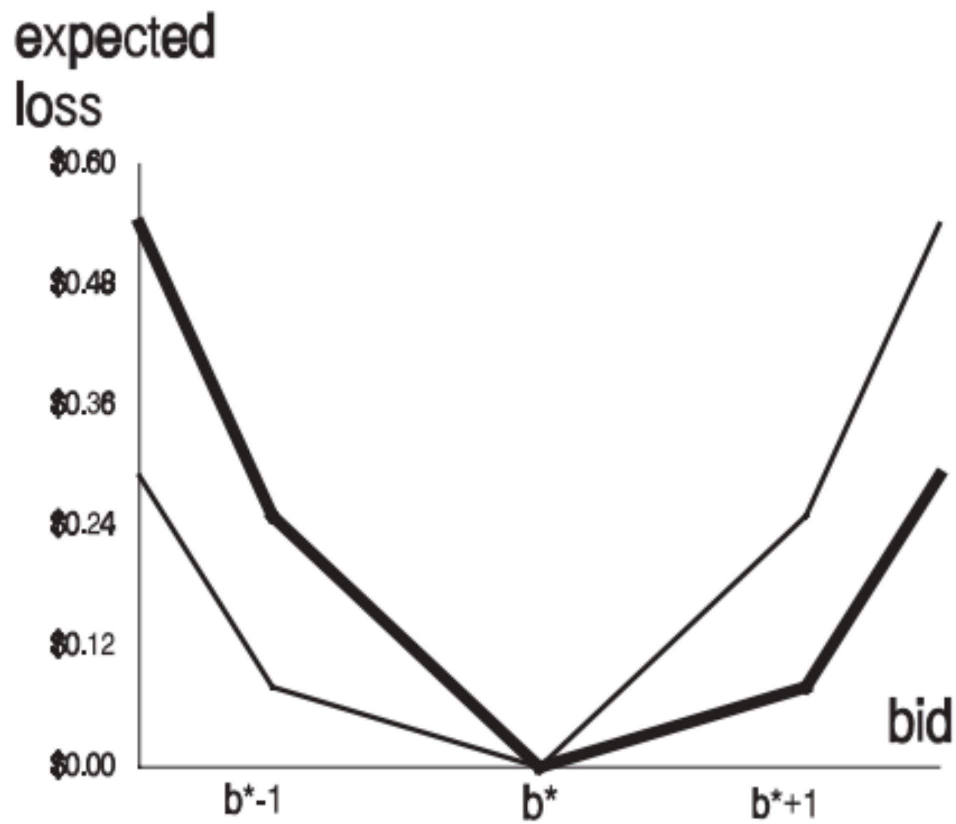


Figure 1. Expected Loss as a Function of Bid for the Two Treatments

QRE and first price auctions

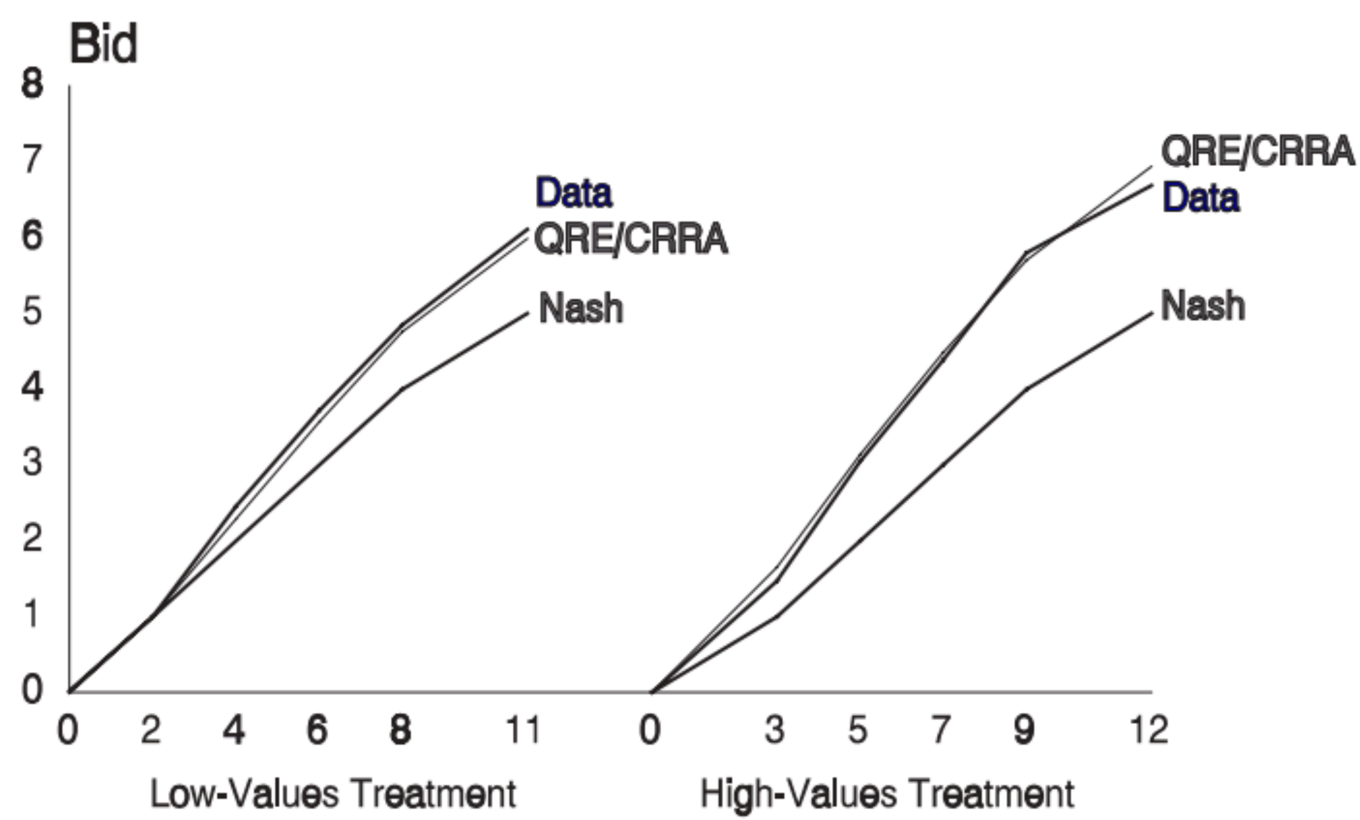


Figure 3. Average Bids By Value For Both Treatments.

QRE and all pay auctions

(Anderson, Goeree & Holt JPE 1998)

- All pay auction: highest bidder gets good, everyone pays bid
  - good model of contests, lobbying etc
- People rarely play equilibrium
  - Mixed strategy eq. where expected payoffs are zero
- QRE predicts
  - Overdissipation of rents
  - rent dissipation to increase with n

Other games with QRE analysis

- Centipede
- Alternating offer bargaining
- Coordination games
- etc
- Camerer, Teck-Hua Ho, and Juin Kuan Chong (2004)
  - *Quantal response equilibrium (QRE), a statistical generalization of Nash, almost always explains the direction of deviations from Nash and should replace Nash as the static benchmark to which other models are routinely compared.*

What happens when people first see a game

- QRE is supposed to model behavior after people understand the game, play for some time but still respond noisily
- What about the first response to a game?
- Start with normal form…

Undercutting game

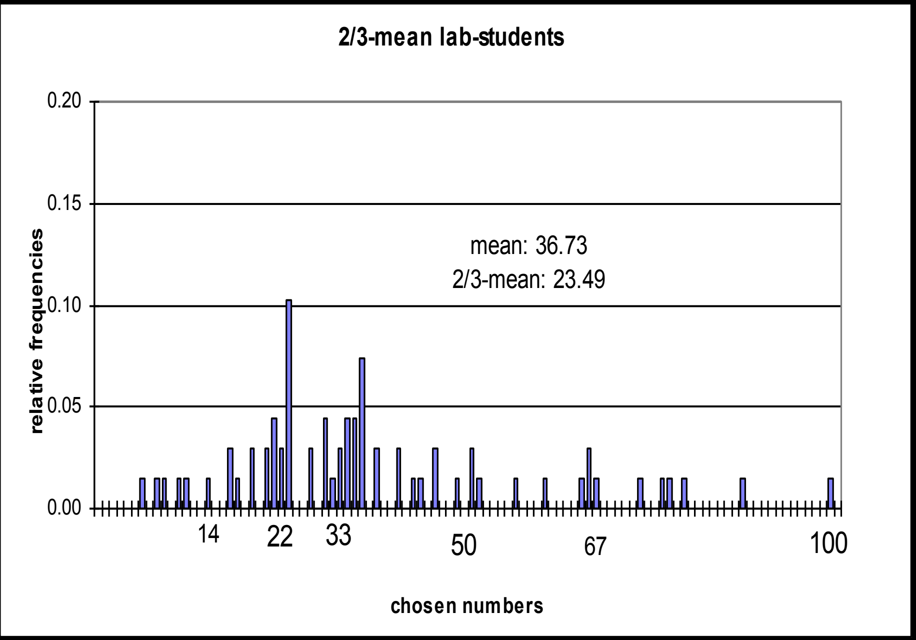
	1	2	3	4	5	6	7
1	1 1	10 -10	0 0	0 0	0 0	0 0	-11 0
2	-10 10	0 0	10 -10	0 0	0 0	0 0	0 0
3	0 0	-10 10	0 0	10 -10	0 0	0 0	0 0
4	0 0	0 0	-10 10	0 0	10 -10	10 -10	10 -10
5	0 0	0 0	0 0	-10 10	0 0	0 0	0 0
6	0 0	0 0	0 0	-10 10	0 0	0 0	0 0
7	0 -11	0 0	0 0	-10 10	0 0	0 0	-11 -11

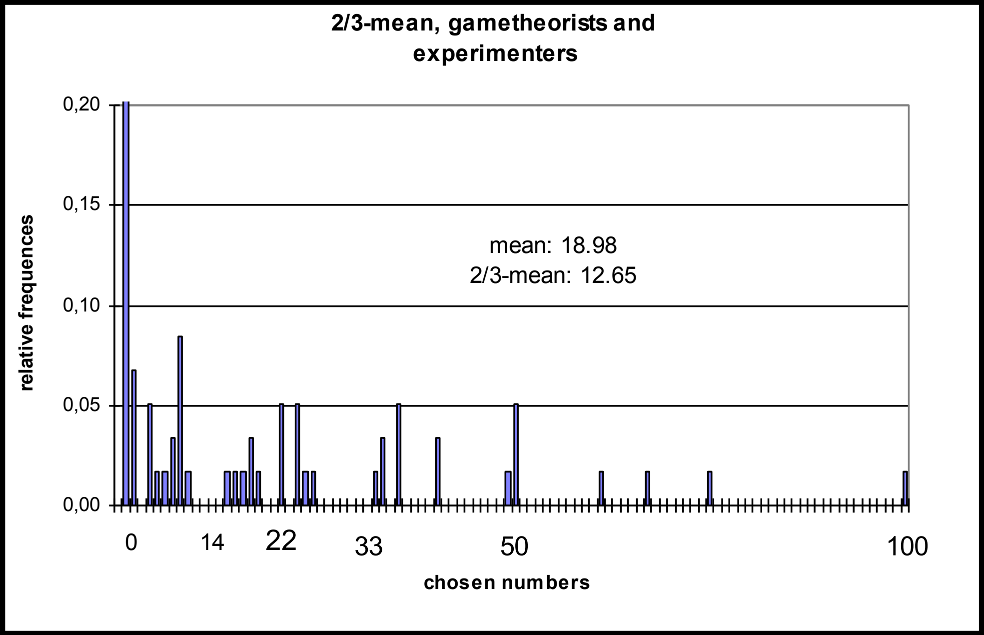
2 person Guessing game

- Two player Games
  - Asymmetric
    - parameterized by a lower bound $a_i \geq 0$, upper bound $b_i > a_i$, and target $p_i > 0$ for each player
  - Strategies are given by $s_i \in [a_i,b_i]$ and player i is paid according to how far her choice is from p_i times s_j, denoted by $e_i = | s_i - p_i s_j |$
    - $15 - (11/200)e_i$, if $e_i \leq 200$
    - $5 - (1/200) e_i$, if $e_i \in (200, 1000]$
    - Zero, if $e_i \geq 1000$
- Example 1: p1 ([100, 500], 0.5) , p2 ([100, 900], 1.3)
- Example 2: p1 ([100, 500], 0.7), p2 ([300, 900], 1.3)

The classic guessing game
(Nagel 1995 AER)

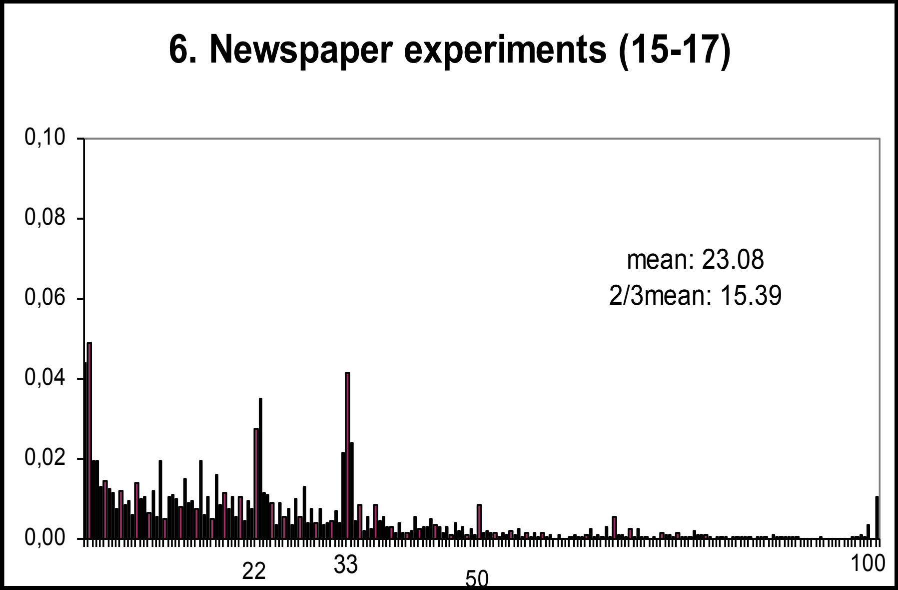
- N players, can say a number [0,100], winner is the person closest to 2/3 times the average number





The classic guessing game

- Theory is very clear: all choose 0
- Actual behavior chaotic?
  - No? there is a structure through the spikes: 22, 33, 0, 67, 100
  - This can be explained by a model of iterated best reply (a concept of game theory)
  - Start assuming others play random
  - L1 best responds to that, L2 to L2 etc

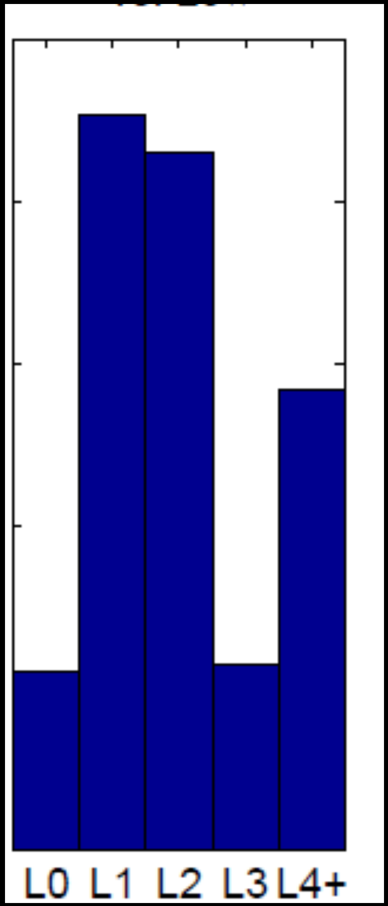


Using models of bounded rationality

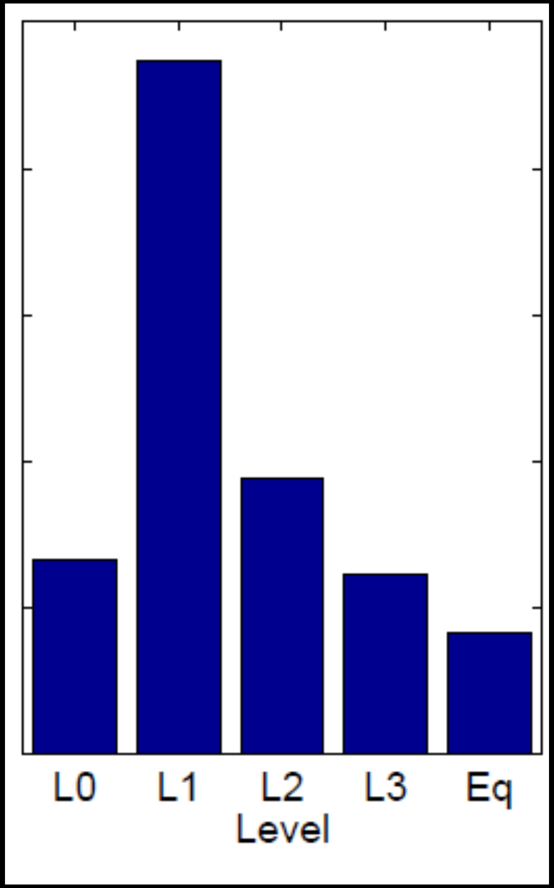
- Level k model also predicts aggregate behaviour well in other lab games
  - 2x2 games, 3x3 games, auctions, hide-and-seek games…
- Level k can even explain the experience of the ECB with liquidity auctions
  - Banks demand liquidity – ECB supplies, if total demand lower than supply there is a proportional rationing rule
  - Nash equilibrium if supply<demand: demand infinity!
  - What actually happened?

Level k Consistence: Aggregate Results

Undercutting games



Guessing games



Are players consistent?

Georganas, Healy and Weber 2011

2 person guessing game results

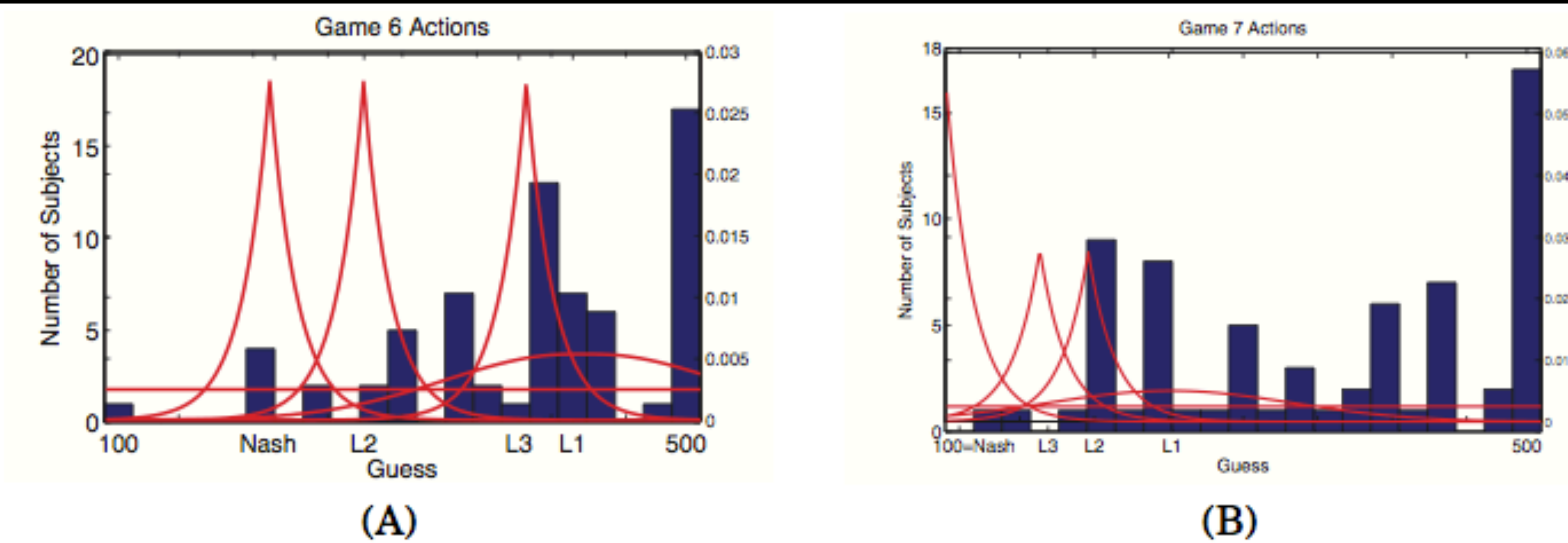


FIGURE VII. Histograms of actions in (A) GG6 and (B) GG7 along with logistic response functions for each level assuming $\lambda = 1$.

Good news?

Bad news?

Switching

From ↓ To →	L0	L1	L2	L3	Nash
L0	8.9%	**48.1%**	17.0%	12.6%	13.3%
L1	13.5%	**50.2%**	18.2%	11.4%	6.7%
L2	11.9%	**45.7%**	24.4%	10.6%	7.3%
L3	13.6%	**44.0%**	16.4%	14.4%	11.6%
Nash	21.2%	**38.2%**	16.5%	17.1%	7.1%
Overall	13.2%	**47.3%**	18.9%	12.3%	8.3%

TABLE V. Markov transition between levels for the six standard two-person guessing games.

Problem Set

1. Find the QRE for the normal form game in slide 9 using the logit specification (for all λ)
   – Algebraically
   – Numerically