LECTURE 7: SINGLE OBJECT AUCTIONS

- Reading
  - Kagel, John H. (1995) Auctions: A survey of experimental results. In: Kagel, John H., Roth, Alvin (Eds.), The Handbook of Experimental Economics. Princeton University press,
- Learning outcomes
  - Know the standard auction types and their equilibria
  - Be familiar with the usual experimental results in these auctions

History

- 500 BC: According to Herodotus women in Babylon were sold through auctions to their potential husbands
  - Starting from the prettiest and proceeding to the least
- 193 AD: the Pretorian Guard sold the entire Roman Empire through an auction
  - Didius Julianus won with 6,250 drachmas per guard
    - Was beheaded two months later – victim of the winner's curse
- 1797 AD: Johann Wolfgang von Goethe sold a manuscript through a second-price auction.
  - Goethe hands a sealed envelope to his notary with his lowest acceptable price to sell and asks his publisher to state a price at which he was willing to buy his manuscript. The publisher would just need to pay the price stated by Goethe in case his price was higher than Goethe's. Otherwise Goethe would not sell his manuscript.
  - Goethe's motivation: to get to know reservation value of publisher
  - Caveat: Publisher got to know Goethe's price through notar
  - **„Goethe's Second-Price Auction" B**enny Moldovanu And Manfred Tietzel, JPE 1998

The present

- Auctions used to sell many assets and goods
- British 3G licence auction in 2000
  - Sold “air”
  - $34 billion, 2.5% of GNP
- Similar auctions in Germany, Holland, Austria, Switzerland
- FCC auctions in the US: Nr. 73 raised $19.6 billion, Nr. 92 $19.8 billion
- Google and Yahoo use auctions for about 90% of their revenues
  - To allocate advertising slots

Who designs auctions?

- “The FCC’s adoption of a simultaneous multiple-round auction ahead of a sequential bid or a single-round auction—which are more conventional but arguably less effective for selling spectrum licenses—was a triumph for game theory. The intriguing next step will be to appraise its performance”
- (p. 160). J. McMillan , Selling spectrum rights. *Journal of Economic Perspectives, Summer* 8 (1994), pp. 145–162.
- So how did things turn out, both on the aggregation front and on the revenue front?
- Also a triumph for experimental economics

Auction types

- Four main types of (single object) auctions studied experimentally
  - English auction (ascending price, real-time)
  - Dutch auction (descending price, real-time)
  - first price sealed bid
  - second price sealed bid
- Procurement version reverses roles: one buyer many sellers.
- Valuations
  - – Independent private values (IPV)
  - – Affiliated private values (APV)
  - – Common values (CV)
- In spectrum market
  - mixture of private and common value
  - simultaneous ascending auction (150 rounds)

Other auction types

- Two sided auction (double oral auction, call markets)
- Sequential auctions (eg. fishmarket)
- multi-unit auctions (FCC auction)
- Auctions with resale opportunities (private value)
- Auctions with toeholds (finance topic)

Why auctions (and not bargaining)?

1. If there are many participants easier with auction
2. Effciency: method most likely to allocate object to the one who values it the most
3. Information revelation: Reveals information about value of object
4. Credibility and transparency
5. Maximize revenue
6. In procurement auction less favouritiss, business plan too time consuming (typically both BP plus auction)
7. Understanding auctions is important given volume of goods traded through auctions

Independent private values sealed bid auction

- N bidders
- One unit of an indivisible good
- Values distributed according to some *f*, say *U [0,100]*
  - Asymmetric info: bidders know own values, no one else does
- Bidders submit bids simultaneousle, can not change them or take them back
- Highest bidder gets object
- Price rule
  - First price: winner pays bid, $\pi_i = v_i - b_i$
  - Second price: winner pays second highest bid, $\pi_i = v_i - \max_{i \neq j}\{b_j\}$

Experiments in veconlab

- Second price IPV
- Common value

Equilibrium: FPA

- Assuming risk neutrality Vickrey (1961) shows that Nash equilibrium bids $b = v\,(n-1)/n$

Consider the following special case:

- Let there be **2 bidders** and let the valuations be independently distributed according to a **uniform distribution** on $[0,V]$ and bidders are **risk neutral**

Proposition: There is a symmetric Nash equilibrium where each bidder chooses the bidding function $b(v)=v/2$

Proof: Let w be the other bidder's valuation, h be his equilibrium bidding strategy and g its inverse

- Then $\Pr(b > h(w)) = \Pr(g(b) > w) = g(b)/V$
- Now assume that h is linear $h(w) = aw$, then
- $g(b)/V = b/aV$, so the expected profit is

  $$(b/aV)(v - b) = (bv - b^2)/aV$$

- Derivative w.r.t. b yields

  $$(v - 2b)/aV = 0 \Rightarrow b = v/2$$

- so the profit maximizing bid is $b(v) = v/2$
- (note this was true for any linear strategy of opponent! Dominant strategy!)

FPA and risk aversion

What happens if bidders are **risk averse**?

- expected profit in FPA is $\Pr(b > h)(v - b)$ (1)
- expected utility is $\Pr(b > h)\, u(v - b)$ (2)
- a risk averse bidder has a concave utility function u, so when maximizing (2) the solution will be at a higher $\Pr(b>h)$ and hence at a higher bid than for (1), i.e. the bidder is more aggressive
- Explanation for lab data?

Results in FPA

Table 4. Theoretical and Pooled Means and Variances: All Auctions

N	Statistic	Dutch, Observed	First, Observed	First, Dutch, Theoretical*	Second Observed	Second Theoretical
3	Mean	2.42	2.44	2.5	1.97	2.5
	Variance	.421	.589	.384	.759	.96
4	Mean	5.33	5.64	4.9		
	Variance	1.63	1.80	.96		
5	Mean	8.78	9.14	8.1		
	Variance	2.06	1.37	1.83		
6	Mean	13.12	13.22	12.1	11.21	12.1
	Variance	3.77	4.31	3.0	8.20	6.4
9	Mean	29.26	31.02	28.9	27.02	28.9
	Variance	7.03	4.91	8.38	18.66	18.85

Note:
* These are the means and variances implied by the Vickrey hypothesis; they are calculated from (3.17)

Results in FPA

- Coppinger et al. (1980) open-outcry; Cox et al.(1982) sequences of first-price/Dutch *clock* auctions.
- Higher prices in the first-price than in the Dutch auctions with the same n, distribution of valuations.
- Greater efficiency in first-price than in Dutch.
  - % of outcomes where high value holder wins.
- Bids are significantly in excess of Risk Neutral Nash Equilibrium (RNNE) bid function predictions for both first-price and Dutch auctions where n>3

Equilibrium in SPA

Proposition: In the 2nd price sealed bid auction it is a (weakly) dominant strategy to bid $b(v) = v$

SPA induces *truth telling*!

Proof: Let h = the highest of the other bids

Assume you bid $b < v$.

- If $h < b$, you win and you pay h.
- But by bidding v, you also win and also pay h.
- If $h > v$, then you do not win with either b or v.
- If $b < h < v$, then with b you do not win, but with v you win and make a profit $v - h > 0$.

Thus $b = v$ **weakly dominates** bidding $b < v$.

Assume you bid $b > v$.

- If $h < v$, then you win and pay h both if you bid b or v.
- If $h > b$, then you do not win in either case.
- But if $v < h < b$, you do not win by bidding v but you win by bidding b and pay h.
  - Your profit then is $v - h < 0$.

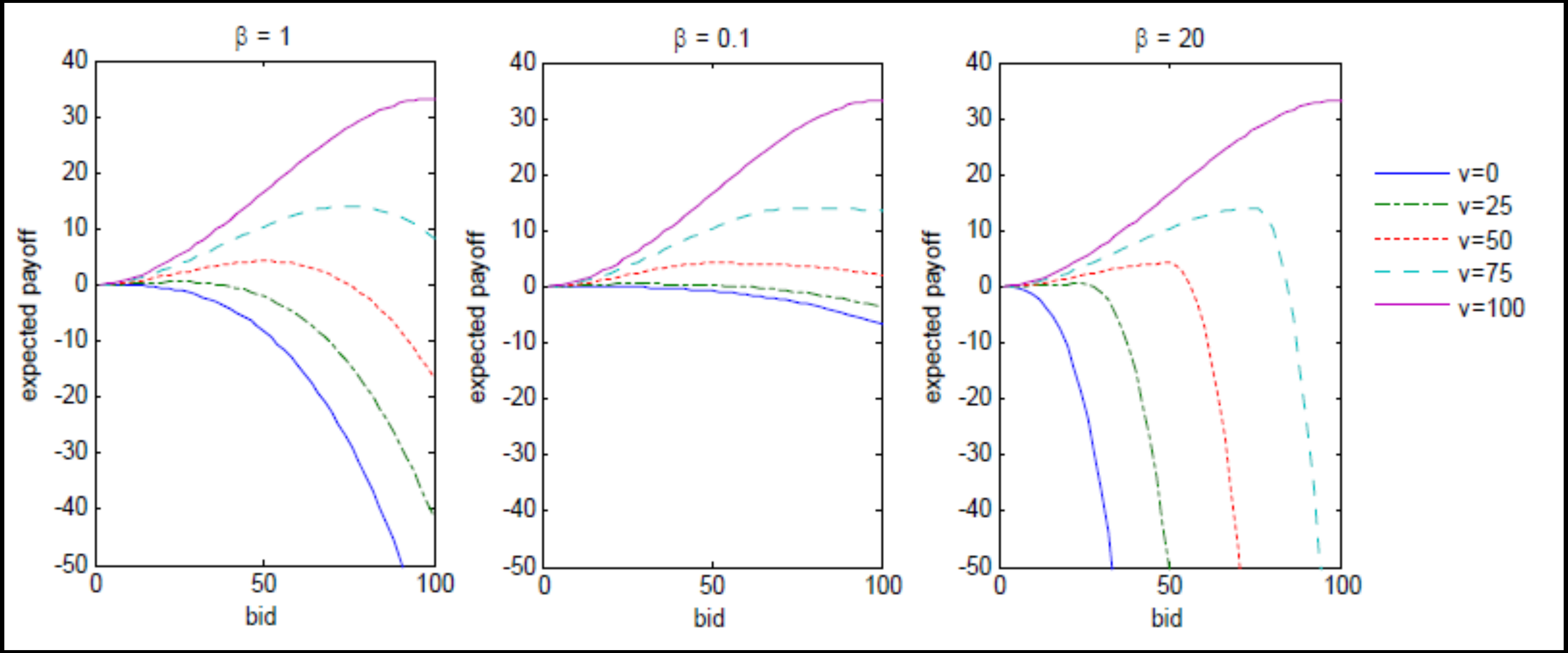
Thus bidding $b = v$ **weakly dominates** bidding $b > v$

Therefore there is a symmetric equilibrium in weakly dominant strategies where each bidder bids his valuation

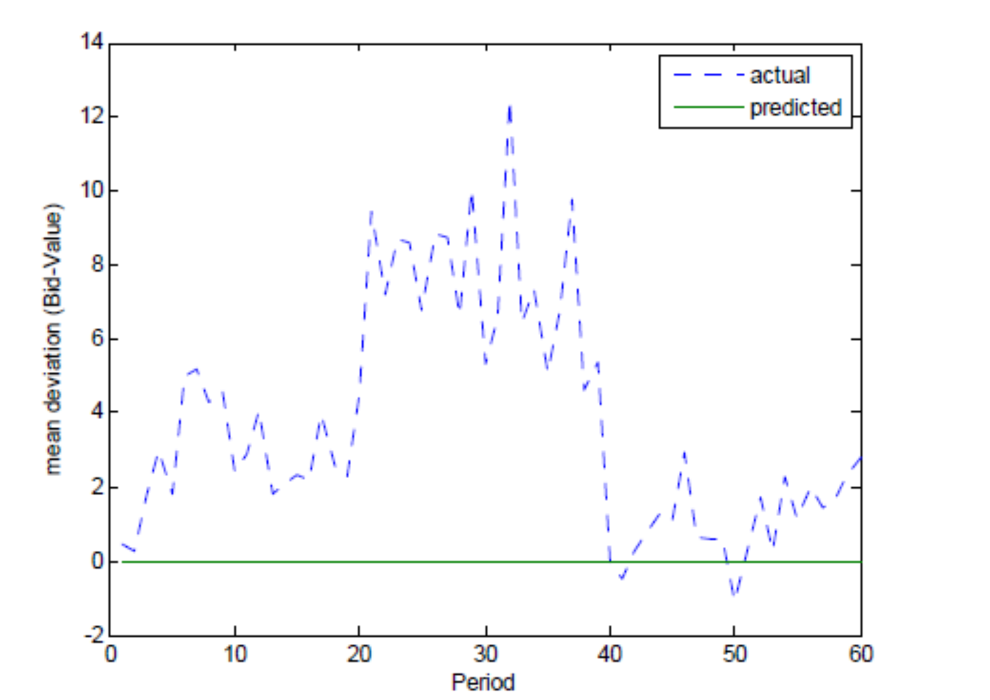
Note that this result is **independent** of the **number** of bidders and the **risk preferences** of the bidders

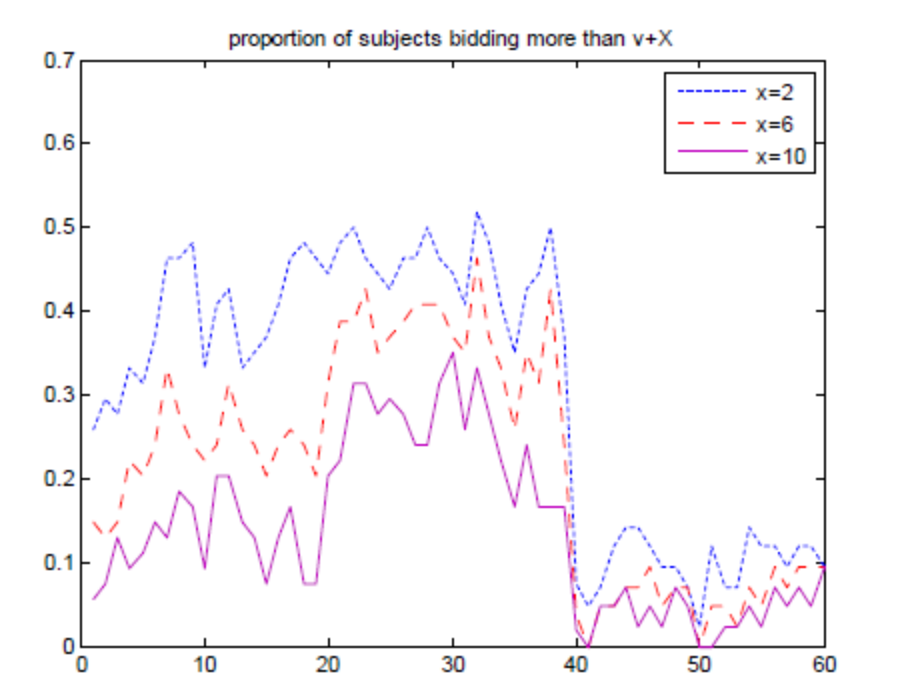
Results in SPA

- Kagel, Levin & Harstad (1987) find systematic overbidding
- Georganas, Levin and McGee (2011) introduce a twist
  - Weigh negative payoffs by some factor b
  - Leaves dominant strategy unchanged, but changes payoff functions

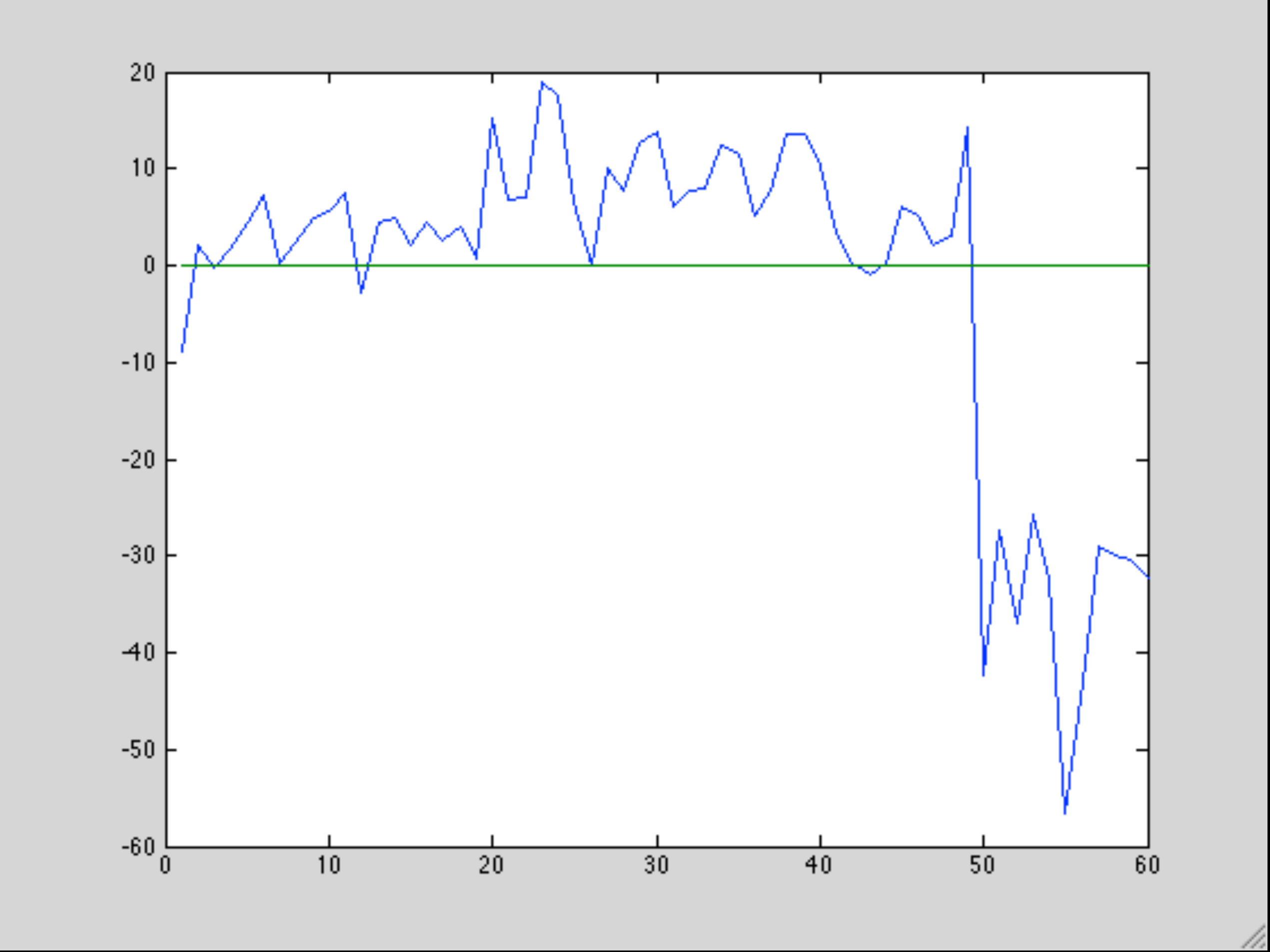


Results in SPA



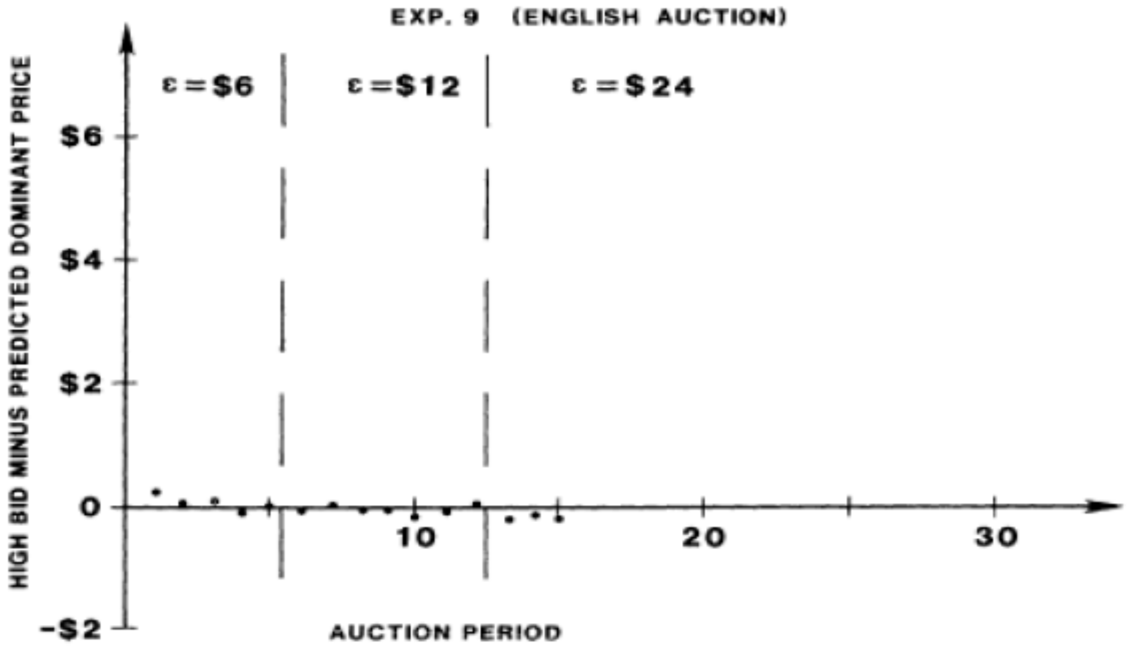


Results in class 29.11.2011



But… bidding of value in strategically equivalent English clock auctions

- Kagel et al. (1987) used affiliated values in their English clock auction (ε is graph below) but the dominance prediction is independent of the nature of valuations.



Affiliated Private Values

- Bidders know their private valuations, but valuations are *positively correlated*, so a high (low) value means others are also likely to have high (low) values.
- Operationally (Kagel et al. (1987)):

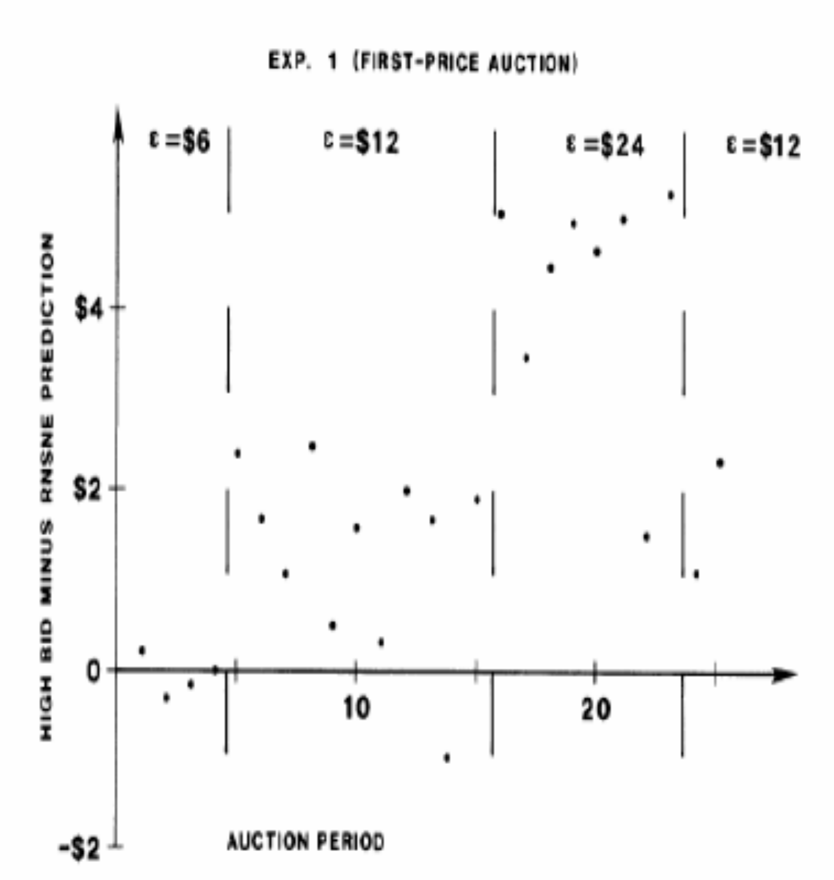
First, draw x_0 from $[\underline{x}, \overline{x}]$.

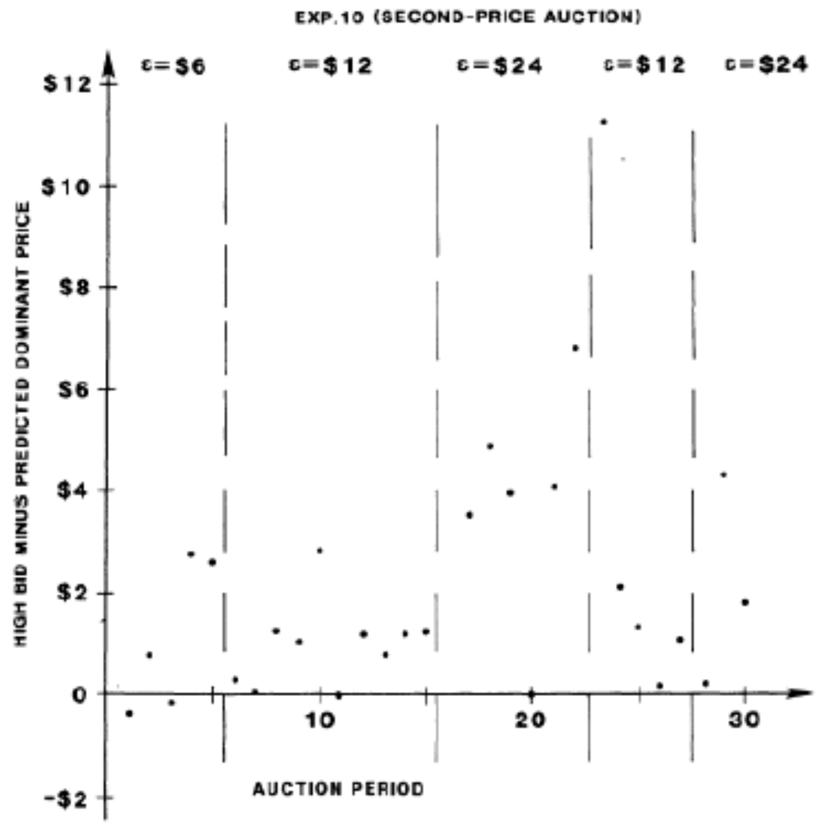
Second, i's valuation x_i is a random draw from $U[x_0 - \varepsilon, \ x_0 + \varepsilon]$.

- If x_0 is unknown: $b_i = x_i - \frac{2\varepsilon}{n} + \frac{Y}{n}, Y = \frac{2\varepsilon}{n+1} e^{-(n/2\varepsilon)[x_i - (\underline{x}+\varepsilon)]}$
- If x_0 is announced, the RNNE bid function is (typically) a generalization of the IPV RNNE bid function: $b_i = \frac{n-1}{n}[x_i - (x_0 - \varepsilon)] + (x_0 - \varepsilon)$

First, Second Price Affiliated Value

- Kagel et al. (1987): x_0 unknown:
- Overbidding observed again and it increases with ε





Common Value Auctions

- Single (common) but unknown value of the object being auctioned (painting, oil field)
- Bidders receive signal values (valuations) affiliated with the common value of the item
- Operationally similar to affiliated values
  1. Draw x_o from $[x_{min},x_{max}]$. This is unknown to players
  2. Bidder i's signal is random draw from x_o+/- ε
- But the bidder's profit is
  - X_0-b1 in first price
  - X_0-b2 in sec price
- If all bidder bid signals, highest signal wins but earns below average or negative payoffs
- The systematic failure to address this adverse selection problem is the *winner's curse*

First Price Sealed Bid CV

- Risk neutral bid function for bidder i with signal x_i (Wilson, Milgrom and Roberts) is:

$$b_i = x_i - \varepsilon + Y\ , \quad Y = \frac{2\varepsilon}{n+1} e^{-n/2\varepsilon[x_i - (\underline{x}+\varepsilon)]}$$

  i.e., presume your signal is the highest and discount accordingly. If all adopt this strategy, the selling price should converge to x_0 for large *n*.
- Subjects are told ε, the support of common values and are informed of the max of $\{x_i-\varepsilon, \underline{x}\}$, min of $\{x_i+\varepsilon, \overline{x}\}$.
- Inexperienced bidders are quite susceptible to the winner's curse in common value auctions, resulting in frequent bankruptcies.
- To deal with bankruptcy, recruit m>n subjects with only *n* non-bankrupt allowed to participate in each period.

$$b_i = x_i - \frac{2\varepsilon}{n} + \frac{Y}{n}, Y = \frac{2\varepsilon}{n+1} e^{-(n/2\varepsilon)[x_i - (\underline{x}+\varepsilon)]}$$

(APV)

Inexperienced bidders: Kagel, Levin, Battalio, Meyer (1989): 11 sessions.
From first 9 auctions (following 2-3 day runs).

Percent of auctions with positive profits:	17.2% (max – 44.4%, min – 0.0%)
Average profits:	-$2.57 (max $0.32, min - $6.57)
Average predicted profits:	$1.90 (max $3.53, min $0.57)
Percent of all bids > $E\left[x_o \middle| X = x_{1n}\right]$	59.4% (max 81.5%, min 35.2%)
Percent of high bids > $E\left[x_o \middle| X = x_{1n}\right]$ (a second type of adverse selection effect)	81.8% (max 100%, min 55.6%)
Percent of subjects going bankrupt	41.1% (max 50.0%, min 16.7%)

Experienced bidders: Kagel and Levin '86

	Small Markets (3-4 bidders)	Large Markets (6-7 bidders)
Actual Profits	\$4.32 (max \$7.53, min \$1.70)	-\$0.54 (max \$1.89, min -\$2.74)
Predicted profits	\$7.48 (Max \$9.51, min \$4.99)	\$4.82 (Max \$5.25, min \$4.70)
Percent of high bids> $E\left[x_o \middle| X = x_{1n}\right]$	19.0% (Max 39.1%, min 0.0%)	53.9% (Max 71.4%, min 22.2%)

Higher individual bids in response to increased number of bidders makes sense in private value auctions, but is the wrong response in common value auctions.

A Theorist's Advice on Avoiding the Winner's Curse (Paul Milgrom)

"Auctions and Bidding: A Primer," *Journal of Economic Perspectives*, 1989.

- "**Mark up your bids twice**: once to correct for the underestimation of costs on the projects you win and a second time to include a margin of profit.
- Don't let the presence of several competing bidders push you into making too aggressive a bid. **The markup to adjust for underestimation will have to be larger the larger is the number of your competitors and the more you respect the accuracy of their cost estimation.**
- **The payoff to careful cost estimation is great**, because it allows you to bid aggressively without great risk.
- If you can also **develop a reputation among your competitors for being an unusually savy estimator**, that's even better for you, because it will compel sensible competitors to bid more cautiously against you and allow you to either increase your profit markup or to win more bids."

Last Minute Bidding on eBay: Field and Laboratory Evidence

- Internet auctions are hybrids of the English and second-price auction formats; proxy-bidding rule.
- eBay hard-close; Amazon soft-close.
- Field evidence: late bidding behavior in hard-close but not soft-close internet auctions (Roth and Ockenfels (*AER* (2002)).

Cumulative freq. of last minute bids

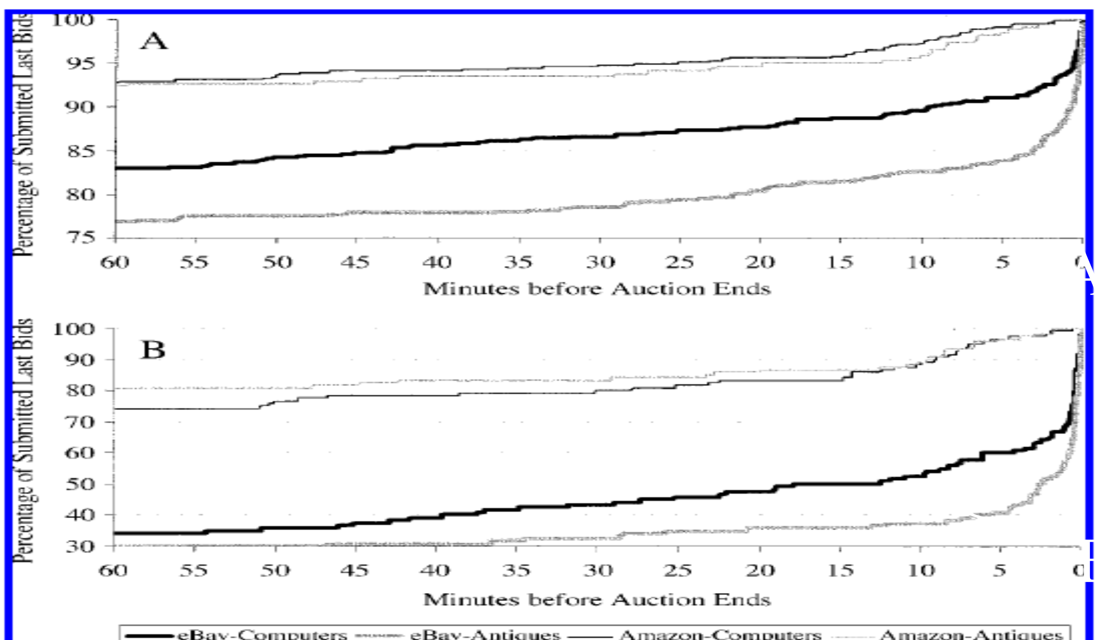


FIGURE 1. CUMULATIVE DISTRIBUTIONS OVER TIME OF (A) BIDDERS' LAST BIDS AND (B) AUCTIONS' LAST BIDS

Problem Set

1. What happens in IPV auctions if resale is allowed? Is bid-your-value still an equilibrium in second price auctions?
2. How would you test if bidders are motivated by spite (they want to reduce the winning bidder's profit)?