Industrial Economics – Sotiris Georganas

R&D Problem Set - Solutions

Problem 1

a) A scientist maximises her expected payoff. The expected per period revenue depending on effort is

$$r = 2\alpha e$$

Given that the patent lasts L years we have $R = Lr$ so the expected utility given effort e

$$EU(e) = L * 2\alpha e - 2e^2$$

Differentiate w.r.t. e

$$\text{FOC: } 2L\alpha = 4e \Rightarrow e = \frac{L\alpha}{2}$$

b) Number of unpatented inventions

$$\chi = q \int_o^{0.5} 2\alpha f(a) da$$

Number of patented inventions

$$\varphi = (1-q) \int_0^{0.5} 2\alpha \frac{L\alpha}{2} f(a) da = (1-q) \int_0^{0.5} 2\alpha \frac{L\alpha}{2} 2da = (1-q) \int_0^{0.5} 2L\alpha^2 da = (1-q)[\tfrac{2}{3}L\alpha^3]_0^{0.5} = (1-q)L\tfrac{2}{24}$$

the planner's utility is then

$$U(L) = 3(\varphi + 4\chi) - L^2 = 3((1-q)L\tfrac{1}{12} + 4\chi) - L^2$$

Differentiating w.r.t. L
(note there is no L in χ so there is no reason to calculate it explicitly)

$$\text{FOC: } L = (1-q)\tfrac{1}{4}$$

c) The higher the number of passionate researchers who are not interested in money, the lower L is. It makes sense, if nobody needed a patent to do research the optimal length L would be 0.

However, L grows in the average ability, which can be seen if you repeat the whole exercise assuming that the average ability is equal to 0.5.

d) No it won't, since scientists have made all the effort already, lowering the patent length does not influence their incentives but it does lower the penalty L^2 in the planner's utility function.

This what we call the time-inconsistency problem of the policy maker, she would like to take back commitments made in previous periods.