# Product Differentiation: Part 1

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# 1 Introduction

- What is the difference between an OPEL (VAUXHALL), a FORD and a FIAT?
- What is the difference between a BMW and a LADA?
- Why do we allow LADAs to be produced or imported?
- What is the optimal number of different products? What is the optimal quality?

- The set of questions we want to address are:

  1. What do we mean by *product differentiation and how can it be represented*?

  2. Does product differentiation *create market power*?

  3. *How much* will firms *choose* to differentiate their products?

  4. What are the *welfare implications* of product differentiation? Does the market overprovide or underprovide variety?

- Contrast with the standard neo-classical model where commodities are exogenous and essentially "unrelated".

# 2 Product differentiation

Definition 1 *The products in an industry are* differentiated *if the consumers view products or brands of various firms as (close but) imperfect substitutes.*

- Example: Toothpaste and shaving cream are *different*, but two brands of toothpaste are *differentiated*.

- In terms of cross price elasticities:

  – Between toothpaste and shaving cream: low (or even zero).

  – Between two brands of toothpaste: significant.

Definition 2 *In a* vertically differentiated product space *commodities differ in quality and all consumers agree on the preference ordering of the commodities.*

Definition 3 *In a* horizontally differentiated product space *the consumers do* not *agree on the preference ordering; if all commodities are sold at the same price the optimal choice depends on the particular consumer.*

# 3 Vertical product differentiation: A model

- Consider a market for a good which can be produced in different qualities.

- Quality is denoted by $s$ and is, by a technological restriction, in an interval $s \in [s_{\mathsf{min}}, s_{\mathsf{max}}]$.

- Suppose that the cost of production per unit of the good is $c$ and is independent of quality.

- All consumers buy one unit of the good, but have different preferences for quality.

- A consumer's preferences be described by

$$\theta s - p. \tag{1}$$

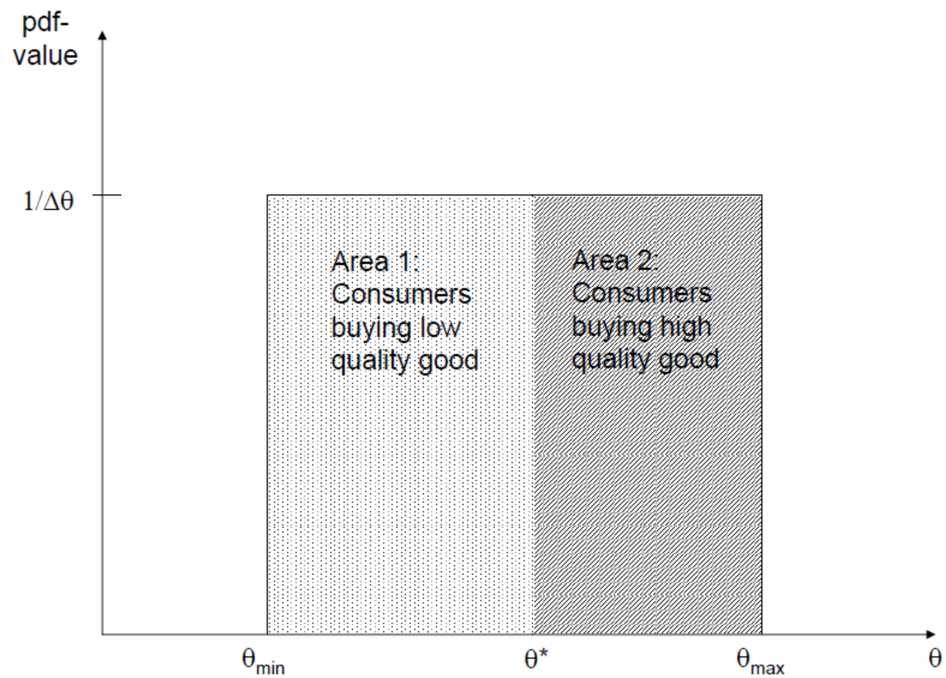
where $\theta$ is the consumer's marginal willingness to pay for quality.

- Heterogeneity of preferences: there is a distribution of $\theta$ among the consumers.

- Assume that $\theta$ is uniformly distributed on an interval $\theta \in [\theta_{\mathsf{min}}, \theta_{\mathsf{max}}]$. Assume $\theta_{\mathsf{max}} > 2\theta_{\mathsf{min}}$ (see below).

- $p$ is the price paid by the consumer.

Fig 1




- Two firms in the market $i = 1, 2$ selling one quality each; firm $i$ sells quality $s_i$.

- Label the firms so that firm 2 sells a higher quality $s_2 \geq s_1$.

- The firms first choose quality, then compete in prices.

*QUESTION*: How do the firms strategically choose their quality levels?

- Second stage of this game amounts to Bertrand competition when commodities are no longer necessarily homogenous.

## 3.1 Price competition

- Solve by backwards induction. Solve in "reverse order"
  - Consumer's demands (given prices and qualities)

- Prices choices by the firms (given qualities)
- Quality choices by the firms

- Clearly if the two firms sell the same quality, then the consumers only base their decision on the price. The interesting case to consider is that where $s_2 > s_1$ and $p_2 > p_1$.

- Consumers with a relatively high willingness to pay for quality will then buy from firm 2 while consumers with a relatively low willingness to pay for quality will buy from firm 1.

- Thus, we can characterize the demand facing each firm by characterizing the critical consumer who is indifferent between the two differentiated products.

- The critical consumer, denoted $\theta^*$ satisfies (see Fig 1)

$$\theta^* s_1 - p_1 = \theta^* s_2 - p_2 \Leftrightarrow \theta^* = \frac{p_2 - p_1}{s_2 - s_1}. \tag{2}$$

- This gives the demand for firm 1 and 2 which equal

$$D_1(p_1, p_2) = \frac{1}{\Delta\theta}(\theta^* - \theta_{\text{min}}) = \frac{1}{\Delta\theta}\left(\frac{p_2 - p_1}{s_2 - s_1} - \theta_{\text{min}}\right) \qquad \text{(Area 1)} \tag{3}$$

$$D_2(p_1, p_2) = \frac{1}{\Delta\theta}(\theta_{\text{max}} - \theta^*) = \frac{1}{\Delta\theta}\left(\theta_{\text{max}} - \frac{p_2 - p_1}{s_2 - s_1}\right) \qquad \text{(Area 2)} \tag{4}$$

where $\Delta\theta \equiv \theta_{\text{max}} - \theta_{\text{min}}$.

*KEY POINT*: By lowering its price, firm $i$ can attract some consumers with a WTP for quality such that they are initially indifferent.

## 3.2 Profits

- Firm $i$'s profits are
$$\pi_i = (p_i - c)\, D_i\,(p_1, p_2)\,. \tag{5}$$
In a Nash equilibrium (of the price setting game) each firm maximizes its own profits given the price set by the other firm (and also given the two qualities $s_1$ and $s_2$).

- Consider the best response function for firm 1; its profits are
$$\pi_1 = (p_1 - c)\,\frac{1}{\Delta\theta}\left(\frac{p_2 - p_1}{s_2 - s_1} - \theta_{\mathsf{min}}\right) \tag{6}$$

- The impact of a marginal increase in its price on profits is

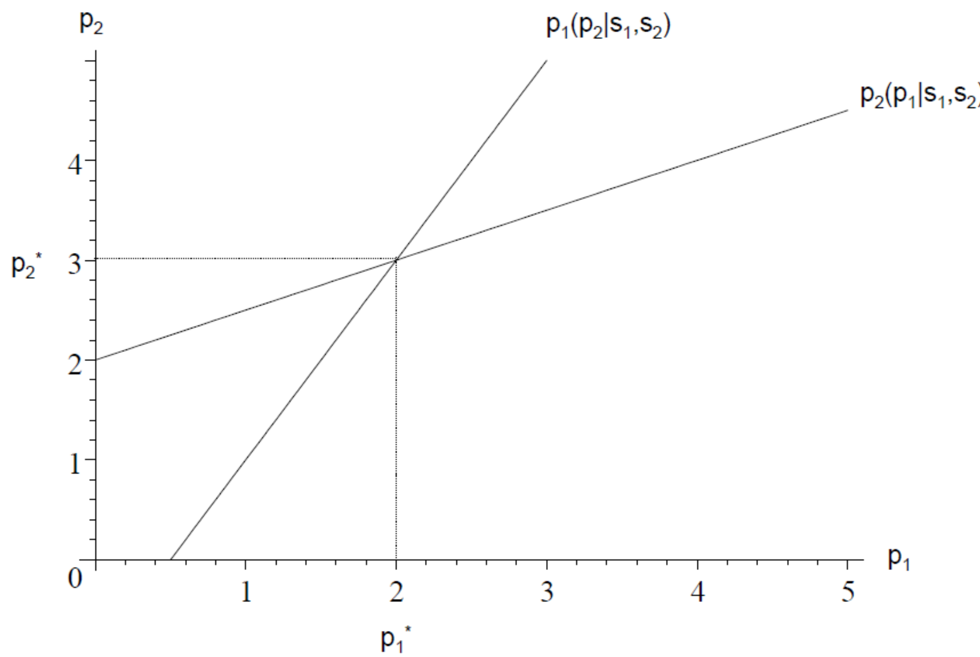
$$\frac{\partial \pi_1}{\partial p_1} = \frac{1}{\Delta\theta}\left(\frac{p_2 - p_1}{s_2 - s_1} - \theta_{\mathsf{min}}\right) - (p_1 - c)\frac{1}{\Delta\theta}\frac{1}{s_2 - s_1} = 0 \qquad (7)$$

- The first component is positive: increasing the "markup", $p_1 - c$, increase the profits from all units sold. However, the second components is negative: increasing the price reduces demand.

- Solving for $p_1$ gives us firm 1's best (price) response

$$p_1(p_2|s_1, s_2) = \frac{c + p_2 - (s_2 - s_1)\,\theta_{\mathsf{min}}}{2} \qquad (8)$$

Fig 2




*KEY POINT*: The best-response functions are *upward sloping*; the higher the price set by firm 2, the high is price optimally chosen by firm 1. It also depends on the quality gap $s_2 - s_1$.

- Similarly, for firm 2, the impact of its price on its profits are

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{\Delta\theta}\left(\theta_{\text{max}} - \frac{p_2 - p_1}{s_2 - s_1}\right) - (p_2 - c)\frac{1}{\Delta\theta}\frac{1}{s_2 - s_1} = 0 \tag{9}$$

- Solving for $p_2$ gives us firm 1's best (price) response

$$p_2\left(p_1 | s_1, s_2\right) = \frac{c + p_1 + (s_2 - s_1)\,\theta_{\text{max}}}{2} \tag{10}$$

- Note that the firms are *not symmetric*: they are selling different quality levels.

- Solving for the Nash equilibrium prices yields

$$p_1^* = c + \frac{(s_2 - s_1)}{3}\left[\theta_{\text{max}} - 2\theta_{\text{min}}\right] \tag{11}$$

$$p_2^* = c + \frac{(s_2 - s_1)}{3}\,[2\theta_{\mathsf{max}} - \theta_{\mathsf{min}}] \qquad (12)$$

*KEY POINT*: Each price is *increasing* in the quality difference $(s_2 - s_1)$. *Quality difference gives monopoly power.*

- How big is the price equilibrium gap?

$$p_2^* - p_1^* = \frac{(s_2 - s_1)}{3}\,(\theta_{\mathsf{max}} + \theta_{\mathsf{min}}) \qquad (13)$$

- Who is the indifferent consumer?

$$\theta^* = \frac{p_2^* - p_1^*}{s_2 - s_1} = \frac{1}{3}\,(\theta_{\mathsf{max}} + \theta_{\mathsf{min}}) \qquad (14)$$

- What are the equilibrium demands?

$$D_1\left(p_1^*, p_2^*\right) = \frac{1}{3\Delta\theta}\left(\theta_{\mathsf{max}} - 2\theta_{\mathsf{min}}\right) \tag{15}$$

$$D_2\left(p_1^*, p_2^*\right) = \frac{1}{3\Delta\theta}\left(2\theta_{\mathsf{max}} - \theta_{\mathsf{min}}\right) \tag{16}$$

- What are the equilibrium profits given the qualities?

$$\pi_1\left(s_1, s_2\right) = \left(p_1^* - c\right) D_1\left(p_1^*, p_2^*\right) = \left(s_2 - s_1\right) \frac{\left(\theta_{\mathsf{max}} - 2\theta_{\mathsf{min}}\right)^2}{9\Delta\theta} \tag{17}$$

$$\pi_2\left(s_1, s_2\right) = \left(p_2^* - c\right) D_2\left(p_1^*, p_2^*\right) = \left(s_2 - s_1\right) \frac{\left(2\theta_{\mathsf{max}} - \theta_{\mathsf{min}}\right)^2}{9\Delta\theta} \tag{18}$$

*KEY POINT*: As long as there is a quality difference $s_2 \neq s_1$ (and $\theta_{\mathsf{max}} -$

$2\theta_{\mathsf{min}}$) both firms make *positive profits*. Indeed, each firm's profits are *increasing in the quality difference*.

- Moreover, the *high quality firm* makes a *larger profit*.

## 3.3 Choice of quality

*QUESTION*: How do the firms choose quality when they anticipate price competition?

- Look for a Nash equilibrium in quality choices.

- The firms will *not* choose the same quality (since this would give zero profit to both firms). Thus one firm will, in equilibrium, provide a strictly lower quality of the good. We have assumed that this is firm 1.

- Given that firm 1 will choose a quality level that is no larger than that chosen by firm 2, its profits increase as the quality $s_1$ is reduced (i.e. as the products are more differentiated).

- Given that firm 2 will choose a quality level that is no less than that chosen by firm 1, its profits increase as the quality $s_2$ is increased (i.e. as the products are more differentiated).

- Thus we conjecture that there will be maximum product differentiation in equilibrium

$$s_1^* = s_{\text{min}}, \quad \text{and} \quad s_2^* = s_{\text{max}}. \tag{19}$$

- Verify: Given $s_2^* = s_{\max}$, $s_1 = s_{\min}$ maximizes $\pi_1$ and, vice versa, $s_1^* = s_{\min}$, $s_2 = s_{\max}$ maximizes $\pi_2$.

- In equilibrium $\pi_2^* > \pi_1^*$: Both firms would like to choose quality first (and then choose the highest quality). There is a gain to being first.

## 3.4 Principle of differentiation and welfare

Definition 4 *The* principle of differentiation*: Firms want to differentiate themselves in order to soften price competition.*

- In general we wouldn't expect to see maximal quality differentiation, there will be opposing forces:
  - If the lowest quality is very low, then no consumer would buy it. Hence a low quality producer faces a trade-off; lowering the quality softens the price competition, but also results in some consumers not buying anything.
  - A firm wants to be where the consumers are: choose the quality level to target large consumer groups.

- Consider now the main questions posed at the beginning of the lecture.
  - Does product differentiation generate market power? Yes – the endogenously differentiated products allow each of the two firms reap positive profits from a set of "loyal" customers.

- How much will the firms differentiate their products? In this "simple" model there will be maximum differentiation.

- What are the welfare implications of product differentiation? Since the marginal cost $c$ is independent of quality (and all consumers appreciate quality), the production of any output of quality less than $s_{\mathsf{max}}$ constitutes an inefficiency and hence a welfare loss.

- In this sense there is *excessive product differentiation*: one firm is choosing to produce an inferior quality good in order to obtain a degree of monopoly power within a segment of the market.

• If the consumer heterogeneity is low (i.e. the spread in the willingness to pay for quality is small, so that $\theta_{\mathsf{max}} \leq 2\theta_{\mathsf{min}}$), then a monopoly outcome will result: the low quality firm cannot catch a market segment.

**Implications for market structure**

- If the consumer heterogeneity is low (in the model $\theta_{\text{max}} \leq 2\theta_{\text{min}}$), monopoly may result: the intense price competition drives any low quality producer out of the market.

- More generally, Shaked and Sutton (1983) showed that even if the production cost $c(s)$ is increasing in quality there can only be a limited number of firms.

- The logic as as above: When firms's products become too similar, this triggers tough price competition which makes entry unprofitable.

# 4 Next Lecture

- In the next lecture we will continue looking at product differentiation, but we will then consider horizontal product differentiation.