Product Differentiation: Part 1

Sotiris Georganas
1 Introduction

- What is the difference between an OPEL (VAUXHALL), a FORD and a FIAT?

- What is the difference between a BMW and a LADA?

- Why do we allow LADAs to be produced or imported?

- What is the optimal number of different products? What is the optimal quality?
The set of questions we want to address are:

1. What do we mean by *product differentiation and how can it be represented*?

2. Does product differentiation *create market power*?

3. *How much* will firms *choose* to differentiate their products?

4. What are the *welfare implications* of product differentiation? Does the market overprovide or underprovide variety?

* Contrast with the standard neo-classical model where commodities are exogenous and essentially “unrelated”. 
2 Product differentiation

Definition 1 The products in an industry are differentiated if the consumers view products or brands of various firms as (close but) imperfect substitutes.

- Example: Toothpaste and shaving cream are different, but two brands of toothpaste are differentiated.

- In terms of cross price elasticities:
  - Between toothpaste and shaving cream: low (or even zero).
  - Between two brands of toothpaste: significant.
Definition 2 *In* a vertically differentiated product space commodities *differ in quality* and all consumers *agree on the preference ordering of the commodities.*

Definition 3 *In* a horizontally differentiated product space the consumers *do not agree on the preference ordering; if all commodities are sold at the same price the optimal choice depends on the particular consumer.*
3 Vertical product differentiation: A model

- Consider a market for a good which can be produced in different qualities.

- Quality is denoted by $s$ and is, by a technological restriction, in an interval $s \in [s_{\text{min}}, s_{\text{max}}]$.

- Suppose that the cost of production per unit of the good is $c$ and is independent of quality.

- All consumers buy one unit of the good, but have different preferences for quality.
A consumer's preferences can be described by

\[ \theta s - p. \]  \hspace{1cm} (1)

where \( \theta \) is the consumer's marginal willingness to pay for quality.

Heterogeneity of preferences: there is a distribution of \( \theta \) among the consumers.

Assume that \( \theta \) is uniformly distributed on an interval \( \theta \in [\theta_{\text{min}}, \theta_{\text{max}}] \). Assume \( \theta_{\text{max}} > 2\theta_{\text{min}} \) (see below).

\( p \) is the price paid by the consumer.
Two firms in the market $i = 1, 2$ selling one quality each; firm $i$ sells quality $s_i$.

Label the firms so that firm 2 sells a higher quality $s_2 \geq s_1$. 
• The firms first choose quality, then compete in prices.

**QUESTION**: How do the firms strategically choose their quality levels?

• Second stage of this game amounts to Bertrand competition when commodities are no longer necessarily homogenous.

### 3.1 Price competition

• Solve by backwards induction. Solve in “reverse order”
  
  – Consumer’s demands (given prices and qualities)
– Prices choices by the firms (given qualities)

– Quality choices by the firms

• Clearly if the two firms sell the same quality, then the consumers only base their decision on the price. The interesting case to consider is that where $s_2 > s_1$ and $p_2 > p_1$.

• Consumers with a relatively high willingness to pay for quality will then buy from firm 2 while consumers with a relatively low willingness to pay for quality will buy from firm 1.

• Thus, we can characterize the demand facing each firm by characterizing the critical consumer who is indifferent between the two differentiated products.
• The critical consumer, denoted $\theta^*$ satisfies (see Fig 1)

$$\theta^* s_1 - p_1 = \theta^* s_2 - p_2 \iff \theta^* = \frac{p_2 - p_1}{s_2 - s_1}. \quad (2)$$

• This gives the demand for firm 1 and 2 which equal

$$D_1(p_1, p_2) = \frac{1}{\Delta \theta} (\theta^* - \theta_{\min}) = \frac{1}{\Delta \theta} \left( \frac{p_2 - p_1}{s_2 - s_1} - \theta_{\min} \right) \quad (Area \ 1) \quad (3)$$

$$D_2(p_1, p_2) = \frac{1}{\Delta \theta} (\theta_{\max} - \theta^*) = \frac{1}{\Delta \theta} \left( \theta_{\max} - \frac{p_2 - p_1}{s_2 - s_1} \right) \quad (Area \ 2) \quad (4)$$

where $\Delta \theta \equiv \theta_{\max} - \theta_{\min}$.

*KEY POINT*: By lowering its price, firm $i$ can attract some consumers.
with a WTP for quality such that they are initially indifferent.

### 3.2 Profits

- Firm $i$’s profits are
  \[ \pi_i = (p_i - c) D_i(p_1, p_2). \]  
  \[ (5) \]
  In a Nash equilibrium (of the price setting game) each firm maximizes its own profits given the price set by the other firm (and also given the two qualities $s_1$ and $s_2$).

- Consider the best response function for firm 1; its profits are
  \[ \pi_1 = (p_1 - c) \frac{1}{\Delta \theta} \left( \frac{p_2 - p_1}{s_2 - s_1} - \theta_{min} \right) \]
  \[ (6) \]
The impact of a marginal increase in its price on profits is

\[
\frac{\partial \pi_1}{\partial p_1} = \frac{1}{\Delta \theta} \left( \frac{p_2 - p_1}{s_2 - s_1} - \theta_{\text{min}} \right) - (p_1 - c) \frac{1}{\Delta \theta} \frac{1}{s_2 - s_1} = 0 \quad (7)
\]

The first component is positive: increasing the “markup”, \( p_1 - c \), increase the profits from all units sold. However, the second components is negative: increasing the price reduces demand.

Solving for \( p_1 \) gives us firm 1’s best (price) response

\[
p_1 (p_2 | s_1, s_2) = \frac{c + p_2 - (s_2 - s_1) \theta_{\text{min}}}{2} \quad (8)
\]
**KEY POINT:** The best-response functions are *upward sloping*; the higher the price set by firm 2, the higher is price optimally chosen by firm 1. It also depends on the quality gap $s_2 - s_1$. 
• Similarly, for firm 2, the impact of its price on its profits are

\[
\frac{\partial \pi_2}{\partial p_2} = \frac{1}{\Delta \theta} \left( \theta_{\text{max}} - \frac{p_2 - p_1}{s_2 - s_1} \right) - (p_2 - c) \frac{1}{\Delta \theta} \frac{1}{s_2 - s_1} = 0
\]  

(9)

• Solving for \( p_2 \) gives us firm 1’s best (price) response

\[
p_2(p_1|s_1, s_2) = \frac{c + p_1 + (s_2 - s_1) \theta_{\text{max}}}{2}
\]  

(10)

• Note that the firms are not symmetric: they are selling different quality levels.

• Solving for the Nash equilibrium prices yields

\[
p_1^* = c + \frac{(s_2 - s_1)}{3} [\theta_{\text{max}} - 2\theta_{\text{min}}]
\]  

(11)
\[ p_2^* = c + \frac{(s_2 - s_1)}{3}[2\theta_{\text{max}} - \theta_{\text{min}}] \]  

\textit{KEY POINT:} Each price is \textit{increasing} in the quality difference \((s_2 - s_1)\). Quality difference gives monopoly power.

- How big is the price equilibrium gap?

\[ p_2^* - p_1^* = \frac{(s_2 - s_1)}{3} (\theta_{\text{max}} + \theta_{\text{min}}) \]  

- Who is the indifferent consumer?

\[ \theta^* = \frac{p_2^* - p_1^*}{s_2 - s_1} = \frac{1}{3} (\theta_{\text{max}} + \theta_{\text{min}}) \]
• What are the equilibrium demands?

\[ D_1 (p_1^*, p_2^*) = \frac{1}{3\Delta \theta} (\theta_{\text{max}} - 2\theta_{\text{min}}) \]  \hspace{1cm} (15)

\[ D_2 (p_1^*, p_2^*) = \frac{1}{3\Delta \theta} (2\theta_{\text{max}} - \theta_{\text{min}}) \]  \hspace{1cm} (16)

• What are the equilibrium profits given the qualities?

\[ \pi_1 (s_1, s_2) = (p_1^* - c) D_1 (p_1^*, p_2^*) = (s_2 - s_1) \frac{(\theta_{\text{max}} - 2\theta_{\text{min}})^2}{9\Delta \theta} \]  \hspace{1cm} (17)

\[ \pi_2 (s_1, s_2) = (p_2^* - c) D_2 (p_1^*, p_2^*) = (s_2 - s_1) \frac{(2\theta_{\text{max}} - \theta_{\text{min}})^2}{9\Delta \theta} \]  \hspace{1cm} (18)

**KEY POINT:** As long as there is a quality difference \( s_2 \neq s_1 \) (and \( \theta_{\text{max}} \)
both firms make \textit{positive profits}. Indeed, each firm’s profits are increasing in the quality difference.

- Moreover, the \textit{high quality firm} makes a \textit{larger profit}.

\subsection*{3.3 Choice of quality}

\textit{QUESTION}: How do the firms choose quality when they anticipate price competition?

- Look for a Nash equilibrium in quality choices.
The firms will *not* choose the same quality (since this would give zero profit to both firms). Thus one firm will, in equilibrium, provide a strictly lower quality of the good. We have assumed that this is firm 1.

Given that firm 1 will choose a quality level that is no larger than that chosen by firm 2, its profits increase as the quality $s_1$ is reduced (i.e. as the products are more differentiated).

Given that firm 2 will choose a quality level that is no less than that chosen by firm 1, its profits increase as the quality $s_2$ is increased (i.e. as the products are more differentiated).

Thus we conjecture that there will be maximum product differentiation in equilibrium

\[ s_1^* = s_{\min}, \quad \text{and} \quad s_2^* = s_{\max}. \] (19)
Verify: Given \( s_2^* = s_{\text{max}} \), \( s_1 = s_{\text{min}} \) maximizes \( \pi_1 \) and, vice versa, \( s_1^* = s_{\text{min}}, s_2 = s_{\text{max}} \) maximizes \( \pi_2 \).

- In equilibrium \( \pi_2^* > \pi_1^* \): Both firms would like to choose quality first (and then choose the highest quality). There is a gain to being first.

3.4 Principle of differentiation and welfare

Definition 4 *The principle of differentiation:* *Firms want to differentiate themselves in order to soften price competition.*
In general we wouldn’t expect to see maximal quality differentiation, there will be opposing forces:

– If the lowest quality is very low, then no consumer would buy it. Hence a low quality producer faces a trade-off; lowering the quality softens the price competition, but also results in some consumers not buying anything.

– A firm wants to be where the consumers are: choose the quality level to target large consumer groups.

Consider now the main questions posed at the beginning of the lecture.

– Does product differentiation generate market power? Yes – the endogenously differentiated products allow each of the two firms reap positive profits from a set of “loyal” customers.
- How much will the firms differentiate their products? In this “simple” model there will be maximum differentiation.

- What are the welfare implications of product differentiation? Since the marginal cost $c$ is independent of quality (and all consumers appreciate quality), the production of any output of quality less than $s_{\text{max}}$ constitutes an inefficiency and hence a welfare loss.

- In this sense there is excessive product differentiation: one firm is choosing to produce an inferior quality good in order to obtain a degree of monopoly power within a segment of the market.

- If the consumer heterogeneity is low (i.e. the spread in the willingness to pay for quality is small, so that $\theta_{\text{max}} \leq 2\theta_{\text{min}}$), then a monopoly outcome will result: the low quality firm cannot catch a market segment.
Implications for market structure

- If the consumer heterogeneity is low (in the model $\theta_{\text{max}} \leq 2\theta_{\text{min}}$), monopoly may result: the intense price competition drives any low quality producer out of the market.

- More generally, Shaked and Sutton (1983) showed that even if the production cost $c(s)$ is increasing in quality there can only be a limited number of firms.

- The logic as as above: When firms’s products become too similar, this triggers tough price competition which makes entry unprofitable.
4 Next Lecture

- In the next lecture we will continue looking at product differentiation, but we will then consider horizontal product differentiation.