

Product Differentiation: Exercises Part 2

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Problem 1

Consider Hotelling's linear city with endogenous prices and exogenous locations. Suppose, however, that there is only one firm, and that this monopolist is (exogenously) located at the left end point of the interval ($y_1 = 0$). Assume that the consumers' transportation costs are quadratic, $k(x - y)^2$, where $k > 0$ is a parameter, x is the location of the consumer, and y is the location of the firm from which she buys. Also, allow for the possibility that some consumers may prefer not to buy at all (which yields zero utility). Solve for the optimal monopoly price in this model assuming zero production costs.

Problem 2

Consider a Hotelling model with two firms: firm 1 is located at $y_1 = 0$, and firm 2 is located at $y_2 = 1$. Consumers are uniformly distributed along the interval $[0, 1]$. Each consumer wishes to buy at most one unit. The utility of a consumer located at x is

$$V_1 - p_1 - kx^2$$

if he buys from firm 1,

$$V_2 - p_2 - k(1 - x)^2$$

if he buys from firm 2, and 0 if he buys from neither firm. V_i represents the "qualities" of the products offered by firm i , while p_i is the price set by firm i . k is a positive constant. For simplicity, assume that the two firms have zero production costs and that they compete by simultaneously setting prices.

(a) Given p_1 and p_2 , compute the location of the consumer who is just indifferent between the two firms (suppose that the market is covered). Explain the intuition of the expression you got.

(b) Given your answer in (a), write the profit maximization problem of each firm. Solve the problem and derive the best-response function of each firm. Show the two best-response functions in a graph that has p_1 on the horizontal axis and p_2 on the vertical

axis, assuming $V_1 = V_2$ and $k = 1$. Solve for the equilibrium set of prices given that $V_1 = V_2$.

(c) Suppose that firm 1 increases V_1 by an amount $a > 0$ by investing in quality (so that $V_1 = V_2 + a$). What is the resulting change in the best response functions of the two firms? Illustrate your answer with a figure and explain the intuition for the resulting changes. Compute the equilibrium prices after the increase in quality by firm 1.

(d) Is the “strategic effect” of the increase in V_1 beneficial or harmful for firm 1? Would firm 1 be more inclined or less inclined to invest relative to the case where it does not engage in price competition with firm 2? Explain your answer. (See CW, p. 532 for a general discussion of strategic effects.)