Product Differentiation: Exercises Part 1

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Problem 1 (from price discrimination lecture)

Assume a monopolist wants to sell bananas to two groups of consumers, aliens and earthlings, who have different wealth (aliens have larger wealth because they own Mars and Venus) but enjoy bananas equally, deriving a utility u(b). Assume both groups' utility for money is a function v(m) with the second derivative being negative. Aliens and earthlings are dressed exactly the same and look exactly the same, so nobody can distinguish them.

a) Which group has a larger demand for bananas?

b) How much would the monopolist be willing to invest in lobbying to be allowed by the intergalactic government to price discriminate? (verbal answer is enough, a graph can be helpful)

c) What is the condition that the monopolist needs to consider so the two groups buy the proposed packages under price discrimination? What is the condition so they prefer to actually buy bananas than leave the market?

d) Assuming lobbying was successful, how much would the monopolist be willing to invest in an alien detection technology? (again, graph is enough)

Problem 2

Consider the model of vertical differentiation from the lecture. There are two firms with zero production costs and a continuum of consumers whose preferences are of the type $U(p, s, \theta)$. Assume θ is uniform in $[0, 2]$.

a) What is the function f describing the pdf (density) of consumers' preferences?

b) What is the condition that characterizes the marginal consumer? Where is the marginal consumer located on the line, if $U(p, s, \theta) = \theta s - p$?

c) Assume that $p_1 = p_2$.What is the location of the marginal consumer? Does anyone buy from firm 1?

d) Assume that $s_1 = 1$, $s_2 = 2$ and $p_1 = 0.5$, $p_2 = 1$.What is the location of the marginal consumer in this case?

e) What is the demand for firm i's good, depending on any prices p_1, p_2 given $s_1 = 1$, $s_2 = 2$?

f) What are the firms' profit functions, what are the best response functions? What are the equilibrium prices in this price setting game given the produced qualities?

g) Are the firms happy with producing qualities $s_1 = 1$, $s_2 = 2$? Will their profits be larger if firm 1 moves to a lower quality? What happens in equilibrium in the quality choice stage?

Solutions: Problem 1

a) Aliens. Since they have a higher wealth, diminishing marginal utility from wealth and their utility derived from bananas is the same as humans, they are willing to trade off more money than the humans for one banana.

b) Remembering the graph from the lecture, he will be willing to pay 2A+C (given that the package for the humans is chosen optimally) minus the producer surplus he gets from selling to the aggregate market at the monopoly price.

c) If the packages are b_a, m_a for the aliens b_e, m_e for the earthlings we have:

IC: $u(b_a) + v(w_a - m_a) > u(b_e) + v(w_a - m_e)$ and vice versa

IR: $u(b_a) + v(w_a - m_a) > v(w_a)$

d) the proceeds from perfect price disrimination minus the profits he would make in (b) (2A+C)

Solutions: Problem 2

a) We know the cdf must be equal to 1 at point 2 and equal to 0 at point 0. This means $F(x) = 1/2x$ Thus the density $f(x) = 1/2$

b) from the lecture $\theta = \frac{p_2 - p_1}{s_2 - s_1}$

c) $\theta = 0$, the marginal consumer is at point 0, so noone buys from the low quality firm.

d) $\theta = 0.5$

e) $D_1 = 0.5(p_2 - p_1\ )$

$D_2 = 0.5(2 - p_2 + p_1\ )$

f) $\pi_i = p_i D_i$

maximise to get the best responses

$p_1(p_2) = p_2/2$

$p_2(p_1) = \frac{p_1+2}{2}$

in eq $p_1^* = 2/3$

$p_2^* = 4/3$

g) maximum differentiation, they go to the two endpoints of the interval.

To summarise, we have following game:

In t=1 firms choose their quality s_i

In t=2 firms choose their price p_i

In t=3 consumers choose to buy or not

The way to solve such a game in general is by backward induction, as we did in this exercise.

1a) Start by looking what consumers do in the last period. This means we start by finding the marginal consumer θ^* as a function of s and p

1b) Find the total demand D_i

2) Find the equilibrium in the price stage. This means, set up our profit functions given we know what consumers will demand and given the qualities we have chosen in t=1.

Find best responses by maximising profits and find equilibrium prices by finding intersection of BRs (equilibrium prices will be functions of s_1 and s_2).

3)Find equilibrium in quality choice stage, given we know what prices will be set in equilibrium in the next stage, for every quality we choose. This means, we set up the profit functions substituting the prices we get from step 2.

Maximise profits and find the equilibrium quality s_1 and s_2, either by finding an interior solution (equating best responses) or by finding a boundary solution (as in this case): firms go to the maximum and minimum points in the interval.