# Product Differentiation: Exercises Part 1 Sotiris Georganas 

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## Problem 1 (from price discrimination lecture)

Assume a monopolist wants to sell bananas to two groups of consumers, aliens and earthlings, who have different wealth (aliens have larger wealth because they own Mars and Venus) but enjoy bananas equally, deriving a utility $u(b)$. Assume both groups' utility for money is of the type $\mathrm{v}(\mathrm{m})$ with the second derivative being negative. Aliens and eartlings are dressed exactly the same and look exactly the same, so nobody can distinguish them.
a) Which group has a larger demand for bananas?
b) How much would the monopolist be willing to invest in lobbying to be allowed by the intergalactic government to price discriminate? (verbal answer is enough, a graph can be helpful)
c) What is the condition that the monopolist needs to consider so the two groups buy the proposed packages under price discrimination? What is the condition so they prefer to actually buy bananas than leave the market?
d) Assuming lobbying was successful, how much would the monopolist be willing to invest in an alien detection technology? (again, graph is enough)

## Problem 2

Consider the model of vertical differentiation from the lecture. There are two firms with zero production costs and a continuum of consumers whose preferences are of the type $U(p, s, \theta)$. Assume $\theta$ is uniform in [0, 2].
a) What is the function $f$ describing the pdf (density) of consumers' preferences?
b) What is the condition that characterizes the marginal consumer? Where is the marginal consumer located on the line, if $U(p, s, \theta)=\theta s-p$ ?
c) Assume that $p_{1}=p_{2}$. What is the location of the marginal consumer? Does anyone buy from firm 1 ?
d) Assume that $s_{1}=1, s_{2}=2$ and $p_{1}=0.5, p_{2}=1$. What is the location of the marginal consumer in this case?
e) What is the demand for firm i's good, depending on any prices $p_{1}, p_{2}$ given $s_{1}=1$, $s_{2}=2$ ?
f) What are the firms' profit functions, what are the best response functions? What are the equilibrium prices in this price setting game given the produced qualities?
g) Are the firms happy with producing qualities $s_{1}=1, s_{2}=2$ ? Will their profits be larger if firm 1 moves to a lower quality? What happens in equilibrium in the quality choice stage?

