# Price Discrimination: Part 2

Sotiris Georganas

# 1 More pricing techniques

- We will look at some further pricing techniques ...

1. Non-linear pricing (2nd degree price discrimination)

2. Bundling

## 2 Non-linear pricing

Definition 1 *With a* non-linear price *a consumer's total expenditure on a good does* not *increase linearly (proportionately) with the number of unit purchased.*

- Useful when
  1. a monopolist faces unobserved heterogeneity in demand, and
  2. the monopolist can prevent resale (no arbitrage)

- Think of the monopolist as designing offers (or "bundles") – each consisting of a *quantity and a total payment* – for each type of consumer.

## 2.1 A Graphical Representation

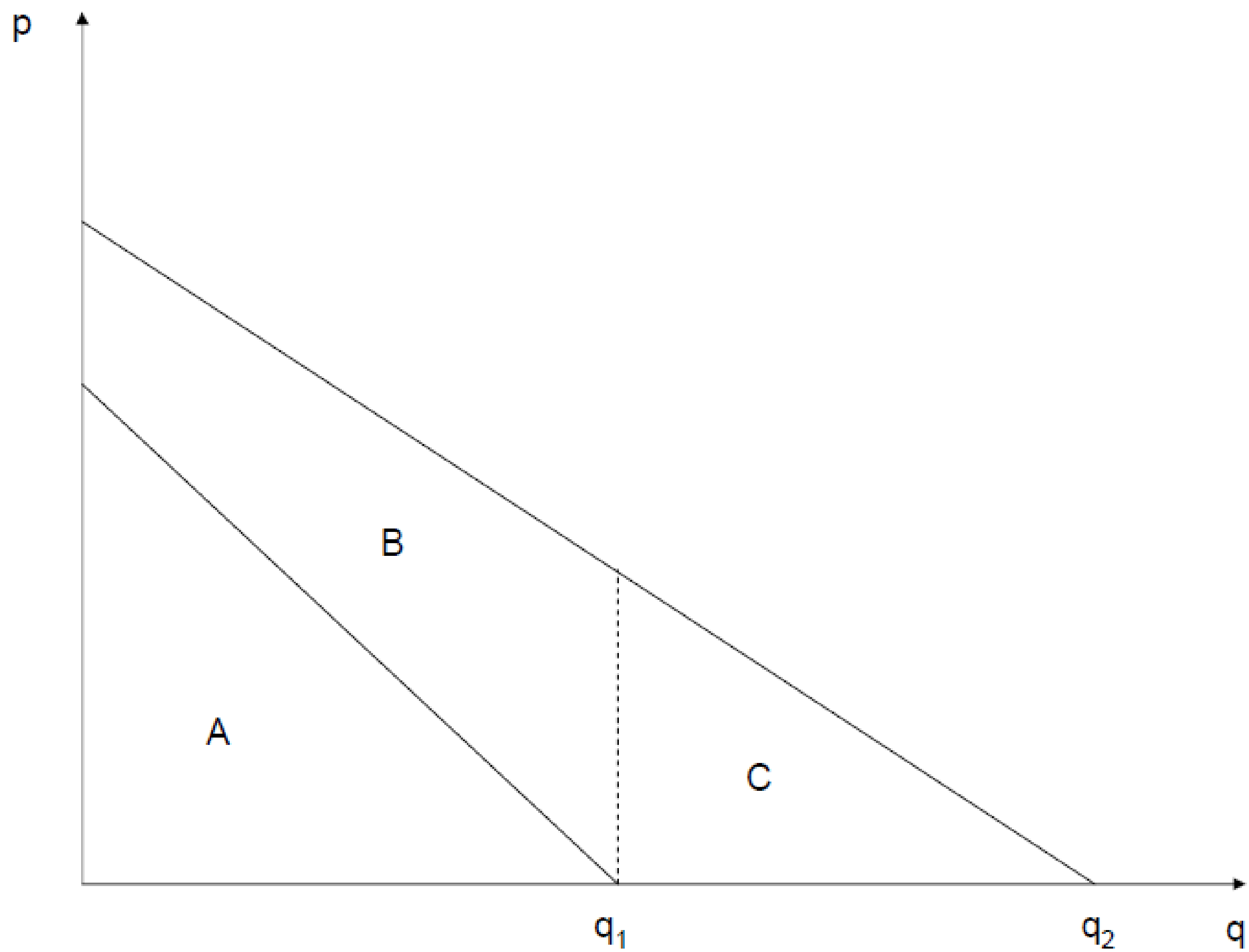
- Consider a monopolist who sells to two types of consumers: high-willingness-to-pay consumers and low-willingness-to-pay consumers. Assume:
  - one consumer of each type.
  - the monopolist cannot tell the consumers apart—he does not know who is the high- and the low-willingness-to-pay consumer respectively.
  - the monopolist's marginal cost is zero (for simplicity): the monopolist maximizes his revenue. The demands for both consumers are displayed in Fig 1.

- If the monopolist *could observe* who is who, he could *price discriminate perfectly*: he would then sell $q_1^0$ to person 1 (the low-demander) at price $A$, and $q_2^0$ to person 2 (the high-demander) at price $A+B+C$. No consumer surplus for the consumers.

- However, this **will not work if the monopolist cannot observe the demand type for each consumer**.

- The high-willingness-to-pay consumer will have an incentive to pretend to be a low-willingness-to-pay consumer; by buying $q_1^0$ at price $A$, he can obtain a surplus equal to $B$.

Fig 1



**What can the monopolist do then?**

- Design price-quantity packages targeted at each type.

- Making sure that each consumer chooses the package intended for her.

- In other words: the consumer are induced to *self-select*: each consumer must prefer the bundle designed for her over that designed for the other type.

- The perfect price discrimination solution does *not* induce self-selection (it is not in high-willingness-to-pay consumer's interest to choose $q_2^0$ at price $A + B + C$).

## A first attempt

- One possibility is to:
  - Offer to sell $q_1^0$ at price $A$, and $q_2^0$ at price $A + C$.
  - The consumers then self-select (the low-willingness-to-pay consumer strictly prefers the low-quantity package; the high-willingness-to-pay consumer is indifferent between the two packages, gaining a surplus of $B$ from each package).
  - Total revenues are $2A + C$ ($A$ from type 1 and $A + C$ from type 2)
- But the monopolist can do even better than that!!!

**The solution**

- The monopolist can reduce the quantity intended for the low-willingness-to-pay consumer from $q_1^0$, at the same time reducing the price.

- This reduces the profit on the low-willingness-to-pay consumer by the small black triangle. Fig 2.

- However, it also makes the low-quantity package *less attractive* for the high-willingness-to-pay consumer.

- The monopolist can then charge more for the high quantity $q_2^0$: the area $C$ has grown.

- At the optimum, $2A + C$ is maximized. Fig. 3.

Fig 2

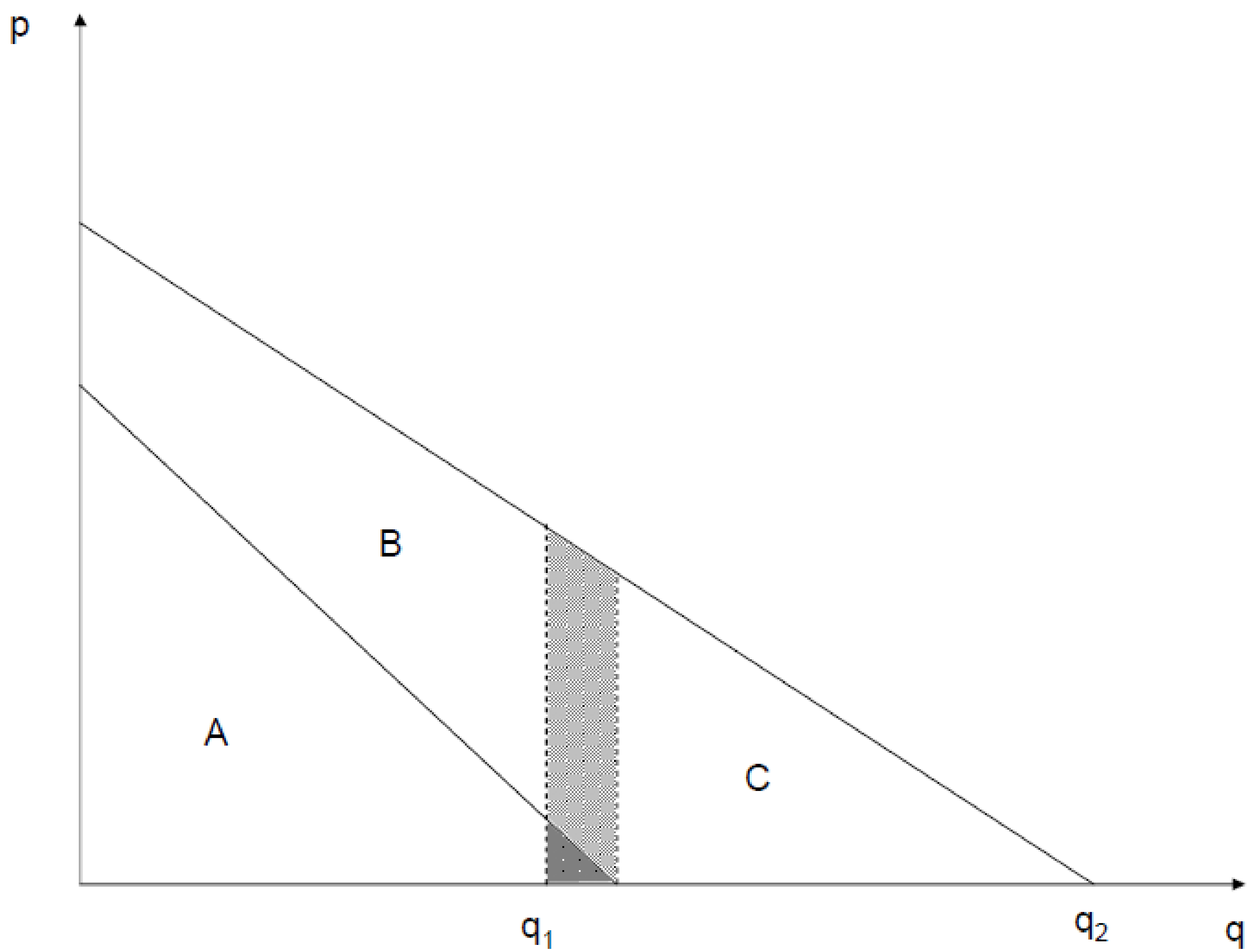
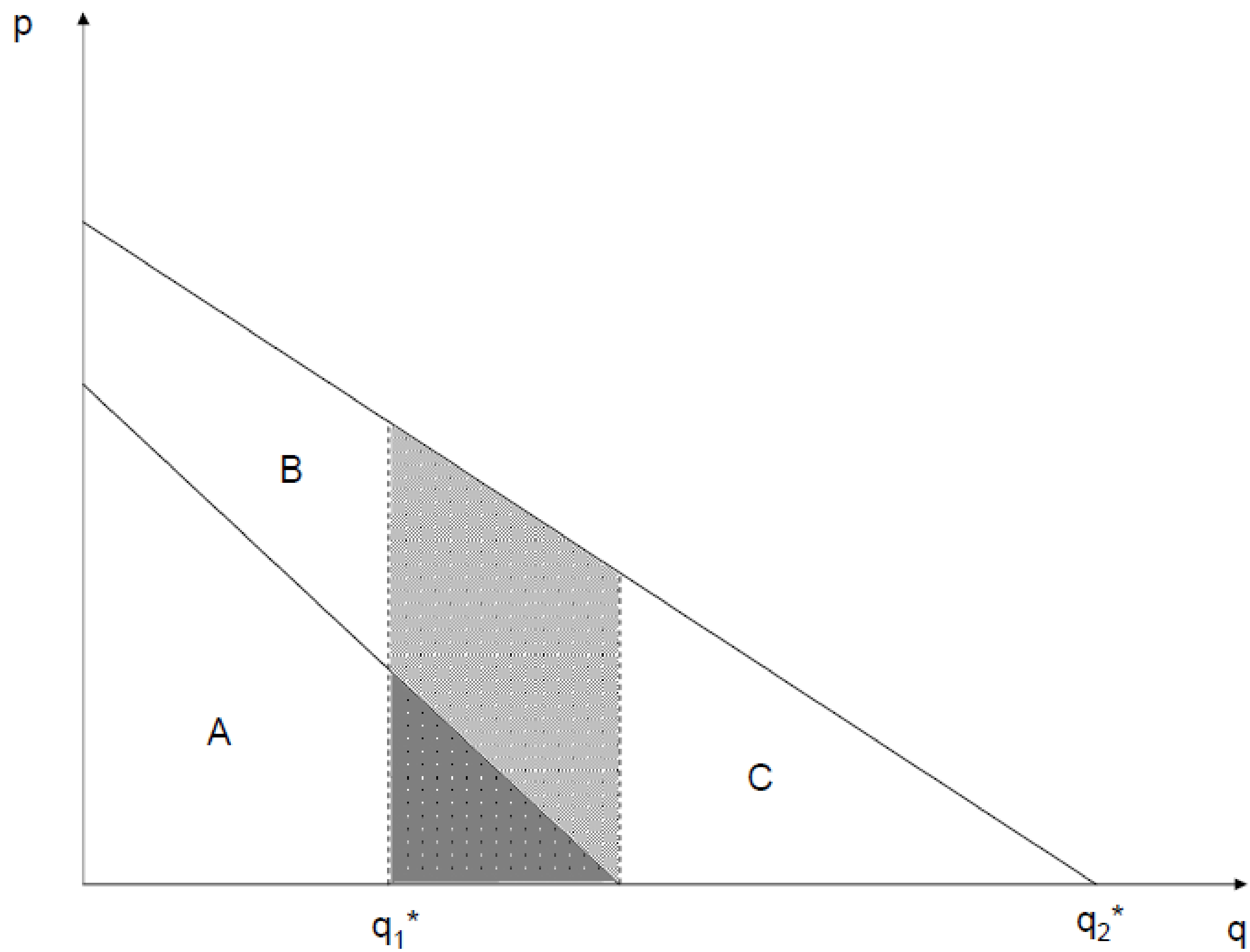


Fig 3



**Properties of the solution**

- The low-willingness-to-pay consumer obtains zero surplus

- The high-willingness-to-pay consumer obtains a positive surplus

- The quantity sold to the low-willingness-to-pay consumer is *lower than the efficient* level $q_1^0$.

- The monopolist offers quantity discounts: the *average price* paid for $q_1^0$ is higher than that for $q_2^0$.

## 2.2 Formal Analysis

- A monopoly firm produces a single good.

- Constant average (and marginal) cost of production: $c$.

- There is a set consumers, each with the following utility:

  $$\theta V(q) - T$$

  when buying a quantity $q > 0$ and paying $T$.

- $V(\cdot)$ is a differentiable function, with $V'(q) > 0$, $V''(q) < 0$ and $V(0) = 0$.

- Hence if I don't buy anything ($q = 0$) and don't pay anything ($T = 0$), my utility is zero.

- $\theta$ is a taste parameter – the strenght of preferences for the good.

- A given consumer's taste parameter $\theta$ is either high or low:

$$\theta \in \{\theta_1, \theta_2\}, \qquad \text{with } 0 < \theta_1 < \theta_2.$$

Assumption 1 *Asymmetric information: The consumer knows his own $\theta$, but the firm does not know it.*

- The firm only knows that a fraction $\lambda$ of all consumers are of type $\theta_1$, and the others (a fraction $1 - \lambda$) are of type $\theta_2$.

- The firm offers two price-quantity bundles to the consumers:
  - $(q_1, T_1)$ is directed to the type-$\theta_1$ consumers.
  - $(q_2, T_2)$ is directed to the type-$\theta_2$ consumers.
- We assume that the parameters are such that it is optimal for the firm to sell to both consumer types, rather than focusing on only the high valuation consumers.

**Analysis**

- The firm chooses $q_1, q_2, T_1$, and $T_2$ so as to maximize its profit,

$$\Pi^m = \lambda (T_1 - cq_1) + (1 - \lambda)(T_2 - cq_2),$$

subject to four constraints:

- Two **individual rationality constraints** and two **incentive compatibility constraints**.
  - Type-$\theta_1$ consumers must prefer their bundle to not trading at all:

$$\theta_1 V(q_1) - T_1 \geq 0. \tag{IR-1}$$

  - Type-$\theta_2$ consumers must prefer their bundle to not trading at all:

$$\theta_2 V(q_2) - T_2 \geq 0. \tag{IR-2}$$

– Type-$\theta_1$ consumers must prefer their bundle to the bundle directed to the type-$\theta_2$ consumers:

$$\theta_1 V(q_1) - T_1 \geq \theta_1 V(q_2) - T_2. \tag{IC-1}$$

– Type-$\theta_2$ consumers must prefer their bundle to the bundle directed to the type-$\theta_1$ consumers:

$$\theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1. \tag{IC-2}$$

- We can simplify the problem by making two observations:
  - **If IR-1 and IC-2 are satisfied, so is IR-2. We can therefore ignore IR-2. To see this, note that**

    $$\text{By IC-2: } \theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1$$

    $$\text{By } \theta_2 > \theta_1\text{: } \theta_2 V(q_1) - T_1 \geq \theta_1 V(q_1) - T_1$$

    $$\text{By IR-1: } \theta_1 V(q_1) - T_1 \geq 0$$

    Hence, completing the chain: $\theta_2 V(q_2) - T_2 \geq 0$, i.e. IR-2 is satisfied.

- **At the optimum, IC-1 is not binding.**
  - We can't ignore IC-1 just like that: we must *prove* that it is not binding.
  - One can do this as follows:

1. Ignore IC-1 and solve for the solution.
2. Check whether IC-1 is satisfied at this solution.
3. If it *is* satisfied, we can conclude that the solution we found must solve also the problem *with* the constraint IC-1.

- The simplified problem: choose $q_1, q_2, T_1$, and $T_2$ so as to maximize

$$\Pi^m = \lambda \left(T_1 - cq_1\right) + \left(1 - \lambda\right)\left(T_2 - cq_2\right), \tag{1}$$

subject to

$$\theta_1 V\left(q_1\right) - T_1 \geq 0 \tag{IR-1}$$

and

$$\theta_2 V\left(q_2\right) - T_2 \geq \theta_2 V\left(q_1\right) - T_1. \tag{IC-2}$$

- Note that since $\Pi^m$ is increasing in $T_1$ and $T_2$, both constraints must bind at the optimum.
  - We thus have
$$T_1 = \theta_1 V(q_1) \tag{2}$$
    and
$$\begin{aligned} T_2 &= \theta_2 V(q_2) - \theta_2 V(q_1) + T_1 \\ &= \theta_2 V(q_2) - \theta_2 V(q_1) + \theta_1 V(q_1) \\ &= \theta_2 V(q_2) - (\theta_2 - \theta_1) V(q_1). \end{aligned} \tag{3}$$

- We can now simplify the problem further, transforming it into an unconstrained problem by substituting in the objective function.

– By plugging (2) and (3) into (1), we get

$$\begin{aligned} \Pi^m &= \lambda \left[\theta_1 V(q_1) - c q_1\right] + (1-\lambda) \times \\ &\quad \times \left[\theta_2 V(q_2) - (\theta_2 - \theta_1) V(q_1) - c q_2\right]. \end{aligned}$$

– The problem now amounts to maximizing this expression for $\Pi^m$ w.r.t. $q_1$ and $q_2$ (without having to take any constraints into account).

- The FOC w.r.t. $q_1$: $\frac{\partial \Pi^m}{\partial q_1} =$

$$\lambda \left[\theta_1 V'(q_1) - c\right] - (1-\lambda)(\theta_2 - \theta_1) V'(q_1) = 0$$

or

$$\theta_1 V'(q_1^*) \left[1 - \frac{(1-\lambda)}{\lambda} \frac{(\theta_2 - \theta_1)}{\theta_1}\right] = c \qquad (4)$$

where the term in brackets is positive but less than one!

- The FOC w.r.t. $q_2$:

$$\frac{\partial \Pi^m}{\partial q_2} = (1-\lambda)\left[\theta_2 V'(q_2) - c\right] = 0$$

or

$$\theta_2 V'(q_2^*) = c \tag{5}$$

**Conclusions**

1. The high-demand consumers buy the socially optimal quantity: **marginal utility/willingness to pay equals marginal cost**. [By (5).]

   - Intuition: Suppose e.g. that the monopolist sold a smaller quantity. Then by increasing the quantity $q_2$ up to the points where type 2's marginal willingness to pay is equal to $c$ and charging the type's willingness to pay for the additional quantity, lead to higher profits but leaves the consumer's utility unaffected.

2. The low-demand consumers buy less than the socially optimal quantity: **marginal utility** $>$ **marginal cost**. [By (4).]

   - Intuition:

- The monopolist wants to extract the high-demand consumer's large surplus.

- An obstacle to this: If the high type gets too little surplus, he can choose the low type bundle instead.

- To prevent this, the monopolist makes the low-type's bundle less attractive by offering those consumers less.

- This works because high-demand consumers suffer more from a reduction in consumption than do low-demand consumers.

• (4) and (5) also imply that $q_2^* > q_1^*$.

3. Low-demand consumers derive **no surplus**, while high-demand consumers derive a **positive surplus**.

4. The relevant personal arbitrage constraint is to prevent high-demand consumers from choosing the low-demand consumers' bundle.

# 3 Commodity bundling

- A pricing strategy available for a monopolist who is selling more than one product.

Definition 2 *A monopolist firm is* bundling *if it offers to sell packages of related goods together.*

- Examples include: (computer software, vacations, films etc.)

- So why bundle?

  – Less expensive to sell together (cheaper production \ distribution)

  – Complementarities in consumption.

## 3.1 Bundling to reduce variability in willingness to pay

- However, we will focus on a third possibility:

Claim 1 *Bundling may be an optimal pricing strategy when there is a high variability in the willingness to pay for the individual goods, but low variability in the willingness to pay for a package of the goods.*

**An example (Varian, Intermediate Microeconomics)**

- A monopolist sell *two* software products: word processor and spreadsheet.

- Marginal cost is zero (for simplicity): revenues are maximized.

- Each consumer buys at most one unit of each product.

- There are two types of consumers: $A$ and $B$ whose willingness to pay are as follows:

**Willingness to pay for products**

| Type of consumer | Word processor | Spreadsheet |
|---|---|---|
| Type $A$ consumer | 120 | 100 |
| Type $B$ consumer | 100 | 120 |

Assumption 2 *No complementaries in consumption – a consumer's willingness to pay for package equals the sum of his willingness to pay for the individual components.*

**Pricing options**

1. Sell each good separately:

   - revenues maximized when each product is sold at £100 (the smallest willingness to pay)

   - total revenues are then £400.

2. Sell a bundle:

   - each consumer is willing to pay £220 for the bundle: set the price for the bundle at £220

   - Total revenues are £440.

## 3.2 General rule

- If the monopolist cannot price discriminate, he has to set the price for each individual product equal to the lowest willingness-to-pay.

- Thus if there is high variability in the willingness to pay, the price for each individual product will be low.

- If the variability in the willingness to pay for a bundle is low, then the monopolist can charge a fairly high price for the bundle.

- In the above case, there was no variability at all in the willingness to pay for the bundle. In this case, the monopolist can actually replicate perfect price discrimination by using the bundling technique.

# 4 What to remember from this lecture

- What is a non-linear pricing scheme. When might a monopolist use a non-linear pricing scheme.

- Properties of an optimal non-linear pricing scheme (that the low-quantity is held back to make it unattractive to the high-willingness-to-pay consumers, that the low-willingness-to-pay consumers obtain no consumer surplus, but the high-willingness-to-pay consumers do, that the solution involves quantity discounts).

- That a multi-product monopolist may be able to increase its profits by using a bundling strategy when the variability in the willingness to pay for the individual products is high but the variability in the willingness to pay for a bundle is low.