Price Discrimination: Exercises Part 2

Sotiris Georganas

Royal Holloway University of London

Problem 1

An economics journal is considering offering a new service which will send articles to readers by email. There are two types of potential users, students and professors. Let x denote the number of articles requested by a user. The professors have an inverse demand function $p_1(x) = 100 - x$ and the students have an inverse demand $p_2(x) = 80 - x$. The marginal cost of sending articles to users is zero.

(a) Draw the demand functions.

(b) Suppose that the journal can identify the type of use (professor or student). It decides to offer a plan where users can buy a fixed number of articles for a fixed price. What price-quantity combination will the journal offer to each type of user?

(c) Now suppose that the journal cannot observe which type any given user is. The journal continues to offer two packages. Suppose that it offers one package which allows up to 80 articles (intended for students) and one package that allows up to 100 articles (intended for professors). What is the highest price that students will be willing to pay for the 80-article package? What is the highest price that the journal can charge for the 100-article package if it offers the 80-article packages at the highest price the students are willing to pay? In this situation, what is the consumer surplus obtained by a professor?

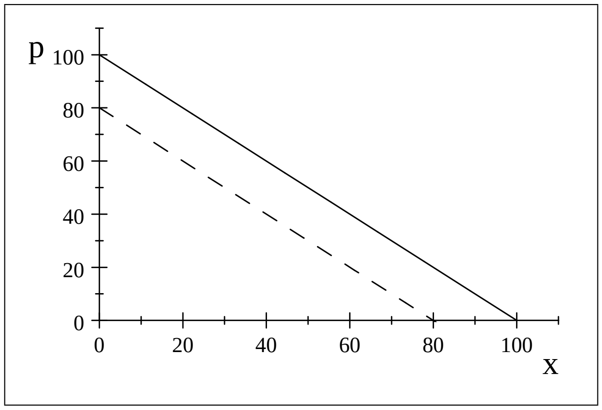
(d) Suppose now that the journal decides to restrict the number of articles in the package intended for the students to 60. What is the most the journal could charge for a 60-article package and still get the students to buy it? How much consumer surplus would the professors get from buying this 60-article package? What is now the most the journal could charge for the 100-article package and still get the professors to choose this package?

(e) Suppose that there is an equal number of students and professors in the population. Would the journal make larger profits by offering the 80-article or the 60-article package to the students?

Solution Problem 1

(a) Start by drawing the demand curves

$$p_1(x) = 100 - x$$



(b) If the journal can identify each user type, then it will first-degree price discriminate. This will involve offering each type of user a package where the quantity is such that the marginal willingness to pay equals the marginal cost. Hence the journal will offer a package that allows up to 100 articles to professors and a package that allows up to 80 articles to students. (In fact, since the marginal cost is zero, the journal could in this case equally well offer unlimited access.) It will charge the professors their entire willingness to pay which is the area under the demand curve from $x = 0$ up to $x = 10$. The size of this area is

$$P_1^* = \frac{100 \times 100}{2} = 5000$$

Similarly, it will charge the students their total willingness to pay for 80 articles

$$P_2^* = \frac{80 \times 80}{2} = 3200$$

(c) If the journal continues to offer the 80-article package, then the highest willingness to pay for this package by students is still $P_2^{**} = 3200$.

In order to work out how much the journal can charge for the 100-article package and still get the professors to choose that package, we first characterize the gross surplus obtained by the professors if they choose the 80-article package.

Note that this gross surplus is the area between the two demand curves between $x = 0$ and $x = 80$. Formally, this area is

$$\int_0^{80} ((100 - x) - (80 - x))\, dx = \int_0^{80} 20 dx = 20 \int_0^{80} dx = 20 \times 80 = 1600$$

Hence, the professors will only choose the 100-article package it it offers a gross surplus of at least 1600. We know that the total willingness to pay, by professors, for 100 articles is 5000. Hence if the journal charges

$$P_1^{**} = 5000 - 1600 = 3400$$

then a professor will obtain the same surplus from buying the 80-article package at the price $P_2^{**} = 3200$ as from buying the 100-article package

(d) If the journal restricts the low-quantity package to at most 60 articles, then the most a student would be willing to pay for a package corresponds to the area under the demand curve between $x = 0$ and $x = 60$, i.e.

$$P_2^{***} = \int_0^{60} (80 - x)\, dx = 3000$$

The gross consumer surplus obtained by a professor from buying the low-quantity package corresponds to the area between the two demand curves between $x = 0$ and $x = 60$, i.e.

$$\int_0^{60} ((100 - x) - (80 - x))\, dx = 20 \int_0^{60} dx = 1200$$

The journal can now price the 100-article package at the price that leaves a professor with a gross surplus of 1200. Since a professor's total willingness to pay for 100 articles is 5000, the journal can now price the 100-article package at $P_1^{***} = 3800$.

(e) Suppose now that there is an equal number of students and professors. For simplicity we can think of there being one student and one professor. (If there are N of each, the profits for the N case will just be N times that obtained when $N = 1$.) Moreover, since marginal costs are zero, we simply need to compare the total revenue in each case.

With an 80-article package intended for the students, the student is charged $P_2^{**} = 3200$ while the professor is charged $P_1^{**} = 3400$. Total revenue/profits are then

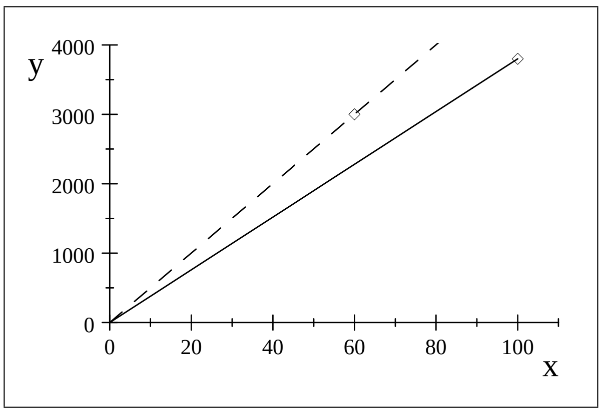
$$P_1^{**} + P_2^{**} = 3400 + 3200 = 6600$$

When the low quantity package is restricted to 60 articles, the journal charges the student $P_2^{***} = 3000$ and charges the professor $P_1^{***} = 3800$. Total revenue/profits are then

$$P_1^{***} + P_2^{***} = 3800 + 3000 = 6800$$

which is obviously higher.

Note the properties of the solution. The quantity sold to the low-demand group – i.e. the students – is distorted downwards. Hence for this group there monopoly generates an allocative inefficiency. For the professors, there is no allocative inefficiency – at the optimum the marginal willingness to pay equals the marginal cost. However, the professors obtain some positive consumer surplus. Note also that the price is non-linear in that the price paid per article is 50 by students while it is only 38 by professors. In effect the monopolist is simply offering a non-linear price scheme from which users can self-select a package. The following figure illustrate the two packages and shows how the average price is lower at the 100-article package than at the 60-article package.



Problem 2

A software company is selling two products: product A (a wordprocessor) and product B (a spreadsheet). The boss of the company is contemplating a marketing strategy that involves bundling the two products together and selling the pair of software products for one price.

Suppose that, at present, the company is selling product A at a price of £200 and it is selling product B at a price of £250. A survey of 100 people who purchased either of these products last year showed that there were three groups of customers.

1. 20 people bought both.

2. 40 people bought only product A; they would have been willing to pay up to £120 for product B.

3. 40 people bought only product B; they would have been willing to pay up to £100 for product A.

Assume that new customers have the same characteristics as the surveyed group. Assume also that the marginal cost of producing extra copies of either product is zero; similarly, the marginal cost of creating a bundle is zero.

(a) Assume that the company offers the products separately as well as bundled. What is the highest price that the company can set for the bundle and still induce group 2 (the word-processor users) to buy it? What is the highest price that the company can set for the bundle and still induce group 3 (the spreadsheet users) to buy it?

(b) What would be the company's profits on a group of 100 users if it priced the bundle at £320? What would be the company's profits on a group of 100 users it priced the bundle at £350? If the company were to offer the bundle, what price should it set?

(c) What would profits be without offering a bundle? Should the bundle be offered?

(d) Suppose that the company worries about the reliability of the survey. It is clear that there are three groups with the above characteristics. However, the company believes that, out of 100 people, t are of type 1 while $(100 - t)/2$ are of type 2 and $(100 - t)/2$ are to type 3. Calculate profits as a function of t assuming that the firm does not bundle. What are profits with the bundle? At what values of t would be be unprofitable to offer the bundle?

Solution Problem 2

It is useful to formalize the problem a bit. Let v_i^j denote the willingness to pay by a consumer in group $i = 1, 2, 3$ for product $j = A, B$. Note first that by buying, the consumers in group 1 reveal they have a willingness to pay for product A that is at least £200 and a willingness to pay for product B that is at least £250; hence we know that $v_1^A \geq 200$ and $v_1^B \geq 250$. Consumers in group 2 are willing to pay $v_2^A \geq 200$ for product A and $v_2^B = 120$ for product B. Consumers in group 3 are willing to pay $v_3^B \geq 250$ for product B and $v_3^A = 100$ for product A. Let $p_A = 200$ denote the price of product A and let $p_B = 250$ denote the price of product B (bought separately). Let p_0 denote the price of a bundle.

(a) Suppose now that the company offers a bundle in addition to selling the products separately. By buying only product A, consumers in group 2 obtain a consumer surplus of

$$v_2^A - p_A = v_2^A - 200.$$

Consumers in this group will prefer to buy the bundle if doing so generates a higher consumer surplus. Hence if

$$v_2^A + v_2^B - p_0 \geq v_2^A - p_A$$

or if

$$p_0 \leq v_2^B + p_A = 120 + 200 = 320.$$

The corresponding calculation for group 3 goes as follows. By buying only product B, consumers in group 3 obtain a consumer surplus of

$$v_3^B - p_B = v_3^B - 250.$$

Consumers in this group will prefer to buy the bundle if doing so generates a higher consumer surplus. Hence if

$$v_3^A + v_3^B - p_0 \geq v_3^B - p_B$$

or if

$$p_0 \leq v_3^A + p_B = 100 + 250 = 350.$$

(b) Suppose now that the company offers the bundle at the price $p_0 = 320$. What product will a consumer in each group buy? Consider a consumer in group 1. This consumer will clearly buy the bundle. We know that a consumer in this group is willing to buy each product when not bundled; hence

$$v_1^A - p_A \geq 0 \text{ and } v_1^B - p_B \geq 0$$

and the total surplus to a consumer from this group from doing so would then be

$$v_1^A + v_1^B - p_A - p_B \geq 0$$

By buying the bundle, the consumer can obtain the surplus

$$v_1^A + v_1^B - p_0 > 0$$

which is a larger surplus since $p_0 = 320 < p_A + p_B = 450$.

Consider a consumer in group 2. This consumer will also buy the bundle. We know that, ignoring the bundle, this consumer will only buy product A leading to the surplus

$$v_2^A - p_A \geq 0.$$

When buying the bundle at the price $p_0 = 320$ the consumer is exactly as well off since, by construction of the price p_0,

$$v_2^A + v_2^B - p_0 = v_2^A - p_A.$$

Finally consider a consumer in group 3. This consumer will also buy the bundle. We know that, ignoring the bundle, this consumer will only buy product B leading to the surplus

$$v_3^B - p_B \geq 0.$$

When buying the bundle at the price $p_0 = 320$ the consumer is strictly as well off since $p_0 = 320$ is less 350 that consumers in this group would, at most, be willing to pay for the bundle. Formally

$$v_3^A + v_3^B - p_0 \geq v_3^B - p_B$$

since

$$p_0 \leq v_3^A + p_B = 350.$$

Hence all 100 consumers choose to buy the bundle at the price $p_0 = 320$ leading to the total revenue

$$\Pi_{p_0=320} = 100 \times 320 = 32\,000$$

Suppose that the price of the bundle was increased to 350. Proceeding as above it is easy to see that consumers in group 1 and 3 would choose to buy the bundle (the latter being indifferent between the bundle and buying only product B); however, since the price $p_0 = 350$ exceeds what the consumers in group 2 would, at most, be willing to pay consumers in group 2 would choose to buy only product A (at the price $p_A = 200$). Hence revenue/profits would be

$$\Pi_{p_0=350} = (20 + 40) \times 350 + 40 \times 200 = \ 29000.$$

Hence we conclude that pricing the bundle at $p_0 = 320$ (and inducing all groups to buy it) generates more profits than pricing at $p_0 = 350$ (and inducing only some consumers to buy it).

(c) Without bundling, consumers in group 1 purchase both products; consumers in group 2 buy only product A while consumers in group 3 buy only product B. Total revenue/profits are then

$$\Pi = p_A (20 + 40) + p_B (20 + 40) = 200 \times (20 + 40) + 250 \times (20 + 40) = 27\ 000$$

Since bundling and pricing the bundle at the optimal price $p_0 = 320$ generates a higher profit, bundling generates more profits.

(d) Now let the group-sizes be

$$N_1 = t,\ N_2 = \frac{(100 - t)}{2},\ \text{and } N_3 = \frac{(100 - t)}{2}$$

with $0 \leq t \leq 100$. Note that above we had that $t = 20$. Now we consider the more general case.

Suppose for that the company does not bundle. Profits are then

$$\begin{aligned}
\Pi(t) &= p_A (N_1 + N_2) + p_B (N_1 + N_3) \\
&= 200 \times \left(t + \frac{(100 - t)}{2} \right) + 250 \times \left(t + \frac{(100 - t)}{2} \right) \\
&= 200 \times \left(50 + \frac{t}{2} \right) + 250 \times \left(50 + \frac{t}{2} \right) \\
&= \left(50 + \frac{t}{2} \right) (200 + 250) \\
&= \left(50 + \frac{t}{2} \right) 450 \\
&= 22\,500 + 225t
\end{aligned}$$

Suppose now that it bundles (at the price $p_0 = 320$). At that price, as shown above, all consumers buy the bundle, leading to the total revenue

$$\Pi_{p_0=320} = 32\ 000.$$

Bundling thus yields a higher profit as long as

$$32\ 000 \geq 22\,500 + 225t$$

or, equivalently, as long as

$$t \leq \frac{32\,000 - 22\,500}{225} \approx 42.2$$

The following figure illustrates revenue/profits as a function of t. The solid line are profits from not bundling. The dashed line are profits from bundling at $p_0 = 320$. (The dotted line are the profits from bundling at the higher price $p_0 = 350$ which is hence not optimal for any value of t.)