Price Discrimination: Exercises Part 1

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Problem 1

A monopolist sells in two markets. The inverse demand curve in market 1 is

$$p_1 = 200 - q_1$$

while the inverse demand curve in market 2 is

$$p_2 = 300 - q_2.$$

The firm's total cost function is

$$c(q_1 + q_2) = (q_1 + q_2)^2$$

The firm is able to price discriminate between the two markets.

(b) What quantities will the monopolist sell in the two markets?

(a) What price will it charge in each market?

Solution Problem 1

This is a straightforward problem which entails setting marginal revenue equal to marginal cost in each market. The only complication is that the total cost function is non-linear implying, an increasing marginal cost. This implies that we have to consider both markets at the same time since e.g. an increase in the output sold in one market increases the common marginal cost relevant to solving the optimal output in the other market. Hence solving the problem will entail solving both market outputs simultaneously; in other words, we will have to solve an equation system.

(a) Note first that the marginal cost is

$$MC = c'(q_1 + q_2) = 2(q_1 + q_2).$$

Next compute the revenue and the marginal revenue from each market. For market 1 we obtain

$$R_1(q_1) = p_1 q_1 = (200 - q_1) q_1 = 200q_1 - q_1^2$$

and hence

$$MR_1 = R_1'(q_1) = 200 - 2q_1.$$

For market 2 we obtain

$$R_2(q_2) = p_2 q_2 = (300 - q_2) q_2 = 300q_2 - q_2^2$$

and hence

$$MR_2 = R_2'(q_2) = 300 - 2q_2.$$

The monopolist will set marginal revenue in each market equal to the (common) marginal cost. Hence, in equilibrium,

$$\begin{aligned} MR_1 &= 200 - 2q_1^* = 2(q_1^* + q_2^*) = MC \\ MR_2 &= 300 - 2q_2^* = 2(q_1^* + q_2^*) = MC \end{aligned}$$

This is an equation system with two equations and two unknown. From the first equation we obtain

$$200 - 2q_1^* = 2(q_1^* + q_2^*)$$

which, solving for q_1^* in terms of q_2^*, yields

$$q_1^* = 50 - \frac{q_2^*}{2}$$

Using this to replace q_1^* in the second equation then yields the following equation in q_2^*

$$300 - 2q_2^* = 2\left(50 - \frac{q_2^*}{2} + q_2^*\right)$$

or

$$300 - 2q_2^* = 100 - q_2^* + 2q_2^*$$

or

$$300 = 100 + 3q_2$$

Solving for q_2^* thus yields

$$q_2^* = \frac{200}{3} \approx 66.67$$

Using this equilibrium value to replace q_2 in the equation for market 1 we then obtain

$$q_1^* = 50 - \frac{q_2^*}{2} = 50 - \frac{(200/3)}{2} = \frac{50}{3} \approx 16.67.$$

Hence, the quantities sold by the monopolist will be $q_1^* = \frac{50}{3}$ and $q_2^* = \frac{200}{3}$.

(b) The equilibrium prices are found simply by plugging the equilibrium quantities into the inverse demand functions. For market 1

$$p_1^* = 200 - q_1^* = 200 - \frac{50}{3} = \frac{550}{3} \approx 183.33$$

while for market 2

$$p_2^* = 300 - q_2^* = 300 - \frac{200}{3} = \frac{700}{3} \approx 233.33.$$

Problem 2

Suppose a supplier can identify two distinct groups of customers, students and non-students. The demand by students q_s and the demand by nonstudents q_n are given by

$$q_s = 100 - 8p_s$$

and

$$q_n = 100 - 4p_n$$

respectively. The total demand, $q_t = q_s + q_n$, is then

$$q_t = 200 - 12p_t$$

The supplier's cost of £2 per unit is constant regardless of the number of units supplied.
(a) What price maximizes profits if the firm charges everyone the same price?
(b) Show that the firm can secure greater profits by charging different prices for the two groups than it can secure by charging everyone the same price.

(c) Graph the demand curves, the marginal revenue curves, the marginal cost curve and highlight the equilibria.

Solution Problem 2

In this case we have somewhat simpler case with a constant marginal cost. This means that, when we consider multi-market price discrimination, the problem simplifies since we can consider each market entirely separately. Moreover, the fact that there is no interaction between the two markets via the marginal cost makes the problem easy to analyze graphically.

(a) For this part the relevant demand is the total demand q_t and the relevant price is the common price p_t. Hence the problem is a straightforward monopoly pricing problem. Since we have been given the demand functions, we can analyze the problem in terms of the price chosen. The monopolist's revenues are

$$R_t = p_t q_t = p_t \left(200 - 12p_t\right)$$

The total costs are

$$C_t = 2q_t = 2\left(200 - 12p_t\right) = 400 - 24p_t$$

Hence the monopolist's profits at price p_t are

$$\pi_t \left(p_t\right) = R_t - C_t = p_t \left(200 - 12p_t\right) - \left(400 - 24p_t\right) = 224p_t - 12p_t^2 - 400.$$

The price is then chosen so as to maximize profits. To find the optimal price, we differentiate the profit function and set the derivative equal to zero,

$$\pi_t' \left(p_t\right) = 224 - 24p_t = 0$$

Solving yields

$$p_t^* = \frac{28}{3} \approx 9.33.$$

Profits at the optimum are given by

$$\pi_t^* = \pi_t \left(p_t^*\right) = 224 \cdot \left(\frac{28}{3}\right) - 12 \cdot \left(\frac{28}{3}\right)^2 - 400 = \frac{1936}{3} = 645.33$$

while the equilibrium total output is

$$q_t^* = 200 - 12p_t^* = 200 - 12 \cdot \left(\frac{28}{3}\right) = 88$$

Above we analyzed the problem directly in terms of the optimal price. We could equally well have solved the problem by solving first for the optimal quantity. To do that, we would work with the inverse demand function; from $q_t = 200 - 12p_t$ we obtain that

$$p_t = \frac{1}{12}\left(200 - q_t\right).$$

Hence we can write revenue as a function of total quantity as

$$R_t = q_t p_t = \frac{q_t}{12}\left(200 - q_t\right) = \frac{1}{12}\left(200q_t - q_t^2\right).$$

This yields the marginal revenue

$$MR_t = \frac{1}{12}\left(200 - 2q_t\right).$$

Hence, setting marginal revenue equal to the marginal cost yields the equation

$$\frac{1}{12}\left(200 - 2q_t\right) = 2$$

which, as above, has the solution $q_t^* = 88$. The implied optimal price for the monopolist is, as above,

$$p_t^* = \frac{1}{12}\left(200 - q_t^*\right) = \frac{1}{12}\left(200 - 88\right) = \frac{28}{3}.$$

Profits are, as above,

$$\pi_t^* = p_t^* q_t^* - 2q_t^* = \left(\frac{28}{3}\right) \cdot 88 - 2 \cdot 88 = \frac{1936}{3} = 645.33$$

For future reference it is also useful to note how much is being sold in each market. Plugging in the optimal common price in the demand functions yields

$$q_s = 100 - 8 \cdot p_t^* = 100 - 8 \cdot \left(\frac{28}{3}\right) = \frac{76}{3} \approx 25.33$$

and

$$q_n = 100 - 4 \cdot p_t^* = 100 - 4\left(\frac{28}{3}\right) = \frac{188}{3} \approx 62.667$$

(b) When the monopolist can set different prices in the two markets it will set the marginal revenue in each market equal to the marginal cost. Solving for the inverse demand yields

$$p_s = \frac{1}{8}(100 - q_s) \quad \text{and} \quad p_n = \frac{1}{4}(100 - q_n)$$

respectively. From this we obtain that the marginal revenues in the two markets are

$$MR_s = \frac{1}{8}(100 - 2q_s) \quad \text{and} \quad MR_n = \frac{1}{4}(100 - 2q_n)$$

respectively. Setting each marginal revenue equal to the marginal cost of 2 yields the following equations

$$\frac{1}{8}(100 - 2q_s) = 2 \quad \text{and} \quad \frac{1}{4}(100 - 2q_n) = 2.$$

Note that, as claimed above, even though we have two markets and hence two equations and two unknowns, the problem simplifies in that each equation can be solved entirely separately. Solving the two equations yields

$$100 - 2q_s = 16 \Rightarrow q_s^* = 42$$

$$100 - 2q_n = 8 \Rightarrow q_n^* = 46$$

Hence, it turns out that total output is still the same, $q_s^* + q_n^* = 88$. However, relative to the case of a common prices, less is now sold in the market with the high demand (i.e. market n) and more is sold in the market with low demand (i.e. market s).

The prices charged in each market can be worked out by plugging the optimal quantities into the inverse demand functions

$$p_s^* = \frac{1}{8}(100 - q_s^*) = \frac{1}{8}(100 - 42) = \frac{29}{4} = 7.25$$
$$p_n^* = \frac{1}{4}(100 - q_n^*) = \frac{1}{4}(100 - 46) = \frac{27}{2} = 13.5$$

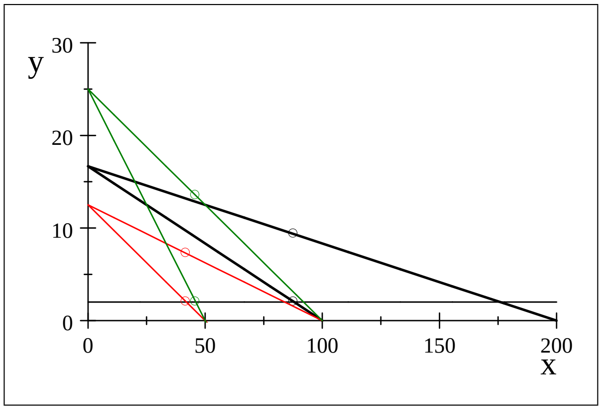
Hence, as expected, a significantly higher price is charged in the market with high demand than in the market with low demand.

We can now verify that the monopolist's profits are higher under multi-market price discrimination. Profits in this case are

$$\pi^{**} = p_s^* q_s^* + p_n^* q_n^* - 2(q_s^* + q_n^*) = \left(\frac{29}{4}\right) \cdot 42 + \left(\frac{27}{2}\right) \cdot 46 - 2 \cdot 88 = 749.5$$

which is indeed higher than the monopolist's profits under a single price (645.33).

(c) The following figure illustrates the problem.



However, it should be noted that we have been somewhat sloppy here. Recall that the total demand at any given price should be $q_t = q_s + q_n$. In other words, it should be the horizonal summation of the demands from the two markets. At prices above $p_t = 12.5$ however, demand from market s is zero. Hence for prices above this level (and up to $p_t = 25$ where demand in market n becomes zero), total demand should formally equal q_t. Hence, to be more correct, we should have drawn the thick total demand curve to coincide with the high market$-n$ demand curve at $p \geq 12.5$. However, luckily, this sloppiness has not invalidated the above answers. In particular, when the monopolist sets a single price he will want to set is low enough that there is positive demand from both markets.

Problem 3

A monopolist has a cost function given by $c(q) = q^2$ and faces an inverse demand curve given by $p(q) = 120 - q$.

(a) What is his profit-maximizing output level? What price will the monopolist charge?

(b) If a lump-sum tax of £100 were put on this monopolist, what would be its profit-maximizing output level?

(c) If you wanted to choose a price ceiling for this monopolist so as to maximize consumer plus producer surplus, what price ceiling should you choose?

(d) How much output will the monopolist produce at this price ceiling?

(e) Suppose that you put a specific tax on the monopolist of £20 per unit of output. What would its profit-maximizing level of output be?

Solution Problem 3

This problem, although less directly on the topic of price discrimination, provides useful insights into the problem of the inefficiencies caused by monopoly pricing.

(a) Revenues as a function of quantity is

$$R(q) = qp(q) = q(120 - q) = 120q - q^2$$

implying that the marginal revenue is

$$MR = R'(q) = 120 - 2q.$$

From the total cost we obtain the marginal cost

$$MC = c'(q) = 2q$$

which is obviously increasing in output.

Profits are maximized at the quantity where $MR = MC$; hence we solve

$$120 - 2q = 2q$$

which yields the monopoly output

$$q^m = 30.$$

The corresponding price is

$$p^m = p(q^m) = 120 - q^m = 120 - 30 = 90.$$

We may also note that profits at the optimum are

$$\pi^m = R(q^m) - c(q^m) = 120q^m - (q^m)^2 - (q^m)^2 = 120 \cdot 30 - (30)^2 - (30)^2 = 1800$$

(b) Given that the tax is lump-sum it shouldn't affect the monopolist's behaviour (only his profits). In particular, neither marginal revenue nor marginal cost is affected by the tax; hence the price and output chosen by the monopolist will be unchanged. The only impact is to reduce the monopolist's profits by the amount of the tax.

(c) To solve this problem we use that consumer plus producer surplus is maximized at the point where price equals marginal cost at it would under competitive pricing. (If this is not clear, then please revisit the notes on monopoly behaviour). It is then clear that a price ceiling can be useful since, in the absence of a price ceiling, the monopolists sets a price that exceeds marginal cost.

The socially optimal price ceiling is thus the one that implements the efficiency rule that price equals marginal cost. Hence to solve for the socially optimal price ceiling we simply set marginal cost equal to price. Recall that $MC = 2q$ while $p = 120 - q$. Setting $MC = p$ yields first the socially optimal quantity; formally solving

$$2q = 120 - q$$

yields

$$q^* = 40.$$

We then use the demand function to obtain the associated price

$$p^* = 120 - q^* = 80.$$

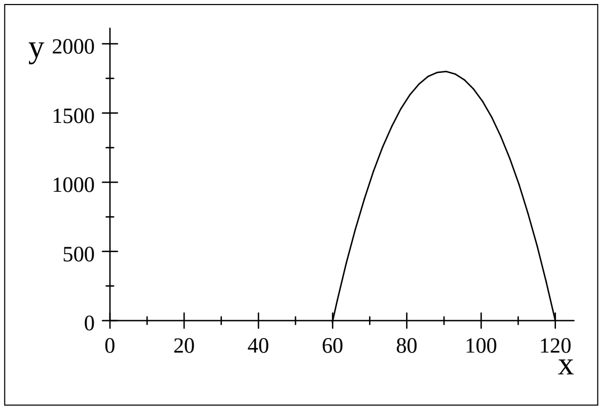
Recalling that the monopolist would like to set the price $p^m = 90$ we see that a price ceiling of $p^* = 80$ indeed has a "bite". We should further note two things: First, by accepting to set the price at the price ceiling, the monopolist is still making positive profits; profits at the price ceiling are

$$\pi^* = p^* q^* - (q^*)^2 = 80 \cdot 40 - (40)^2 = 1600 > 0.$$

which is indeed positive but less than the monopolist's profits in the absence of a price ceiling. Second, setting the price at the price ceiling is the best available option for the monopolist: setting a price strictly below the price ceiling will generate lower profits. To see verify this formally, we can write the monopolist's problem as a price-setting problem. The demand function is $q(p) = 120 - p$. Hence the monopolist's profits as a function of the price p are

$$\pi(p) = pq(p) - c(q(p)) = p(120 - p) - (120 - p)^2 = 360p - 2p^2 - 14\,400$$

Plotting profits as a function of price yields the following figure.



This shows that the monopolist's (unconstrained) optimal price is indeed $p^m = 90$. Moreover, if the monopolist is constrained to setting $p \leq 80$, then accepting the price ceiling by setting $p = 80$ generates the highest (constrained) profits.

(e) As in (a) MR=120-2q.

MC=2q+20 Equating the two we get 4q=100 or q=25