# UNIVERSITY OF LONDON

# BSc EXAMINATION 2013

For Internal Students of
Royal Holloway

# MODEL ANSWERS

## EC3313: Industrial Economics - Spring Midterm

Time Allowed: ONE hour

Answer ALL questions .

# MODEL ANSWER COPY

1. Consider a monopolist who sells two products, towels and face cloths, to two consumers. The marginal cost of each product is, for simplicity, zero. The consumers differ in tastes; consumer 1's willingness to pay is \$4 for a towel and \$1 for a face cloth, while consumer 2's willingness to pay is \$3 for a towel and \$2 for a face cloth. Each consumer's willingness to pay for a bundle containing a towel and a face cloth is simply the sum of the willingness to pay for the separate products. Should the monopolist bundle the products instead of selling them separately in order to maximize profits? Show your calculations and motivate your answer. In particular explain when bundling makes sense in general. Note: the answer does not have to be long, as long as it is precise and clear. **(10 marks)**

   **Model Answer**

   Let $p_1$ denote the price set for towels (product 1) and let $p_2$ denote the price set of face cloths (product 2). Let $v_i^j$ denote consumer $i$'s willingness to pay for product $j$. We have been given

   $$v_1^1 = 4, v_1^2 = 1$$
   $$v_2^1 = 3, v_2^2 = 2$$

   Consider selling the products separately. Consider first product 1. What is the revenue/profit maximizing price? Demand is $q_1 = 2$ for $p_1 \leq 3$, $q_1 = 1$ for $3 < p_1 \leq 4$, and $q_1 = 0$ for $p_1 > 4$. Hence we see that revenue is maximized by setting $p_1 = 3$, leading to the maximum revenue of $p_1^* q_1^* = 6$. For product 2, demand is $q_2 = 2$ for $p_1 \leq 1$, $q_1 = 2$ for $1 < p_1 \leq 2$, and $q_1 = 0$ for $p_1 > 2$. Hence the maximum revenue is 2, which is obtained either at the price $p_1 = 1$ (with $q_1 = 2$) or at the price $p_1 = 2$ (with $q_1 = 1$). Hence total revenue when not bundling is $6 + 2 = 8$.

   Now, consider bundling. The most each of the two customer is willing to pay for a bundle is $v_i^1 + v_i^1 = 5$. Hence if the monopolist offers bundles at the price of 5, then both customers buy the bundle and total revenue is 10. Hence the monopolist should bundle. The intuition is simple: bundling has eliminated the variation in willingness to pay. Bundling in general is attractive when the willingness to pay for individual products has a higher variance than the willingness to pay for a bundle.

2. In 1998, the European Commission fined Volkswagen more than 100m euros for preventing its dealers in Italy from selling to foreign buyers. Knowing that Volkswagen is using price discrimination, is this the right decision from a Pan-European social welfare point of view? A concise answer should be enough, you can use graphs if you need to. **(10 marks)**

   **Model Answer**

   It depends, of course. When you allow foreign sales, you are practically merging different markets. There is the possibility that prohibiting different prices in different markets will lower welfare, for example if one of the markets was weak/small and the price in the merged market will exclude all clients of the small market from buying.

3. Consider two firms whose products are imperfect substitutes. The per-period demand for each firm's product depends, in part, on the price that the rival charges for its product. Specifically, suppose that

$$q_1 = 16 - 4p_1 + 2p_2$$

and

$$q_2 = 16 - 4p_2 + 2p_1$$

Suppose that both firms have zero marginal costs.

**(a)** What are the prices set by the two firms in a (static) equilibrium where the firms compete in prices? What is the level of profits obtained by each firm?

**(b)** Suppose now that the two firms collude by coordinating their prices. What common price $p$ should they agree on in order to maximize total joint profits? What are the resulting profits?

**(c)** Suppose now that firm 1 deviates from the collusive price. Determine the firm's optimal deviation and its profits.

**(d)** Suppose now that the horizon is infinite and that the firms discount the future by the factor $\delta$¡1. For what values of $\delta$ is a grim trigger collusive agreement a subgame perfect Nash equilibrium? (tip: every player follows a grim-trigger strategy where a deviation from any firm from the collusive agreement leads to the Nash equilibrium derived in part (a) being played forever after) **(30 marks)**

**Model Answer**

Given the prices $(p_1, p_2)$ profits for firm $i$ is

$$\pi_i (p_1, p_2) = p_i q_i = p_i (16 - 4p_i + 2p_j)$$

where firm $j$ is the "other" firm. Firm $i$ chooses the price $p_i$ to maximize $\pi_i$ given price $p_j$. The first order condition satisfied by the optimal $p_i$ is

$$(16 - 4p_i + 2p_j) - 4p_i = 0$$

It would be fine here to assume symmetry, set $p = p_i = p_j$ and proceed to solve for p.

More formally however:

Solving for $p_i$ yields the best-response function

$$p_i (p_j) = 2 + \frac{p_j}{4}$$

It is easy to see that there will be a unique symmetric equilibrium. Formally, a price equilibrium is a pair $\left(p_1^*, p_2^*\right)$ where $p_1^*$ is a best-response by firm 1 to firm 2 setting price $p_2^*$ and vice versa. Hence, formally, we have the equation system

$$p_1^* = 2 + \frac{p_2^*}{4} \text{ and } p_2^* = 2 + \frac{p_1^*}{4}$$

Using the latter equation to substitute for $p_2^*$ in the first we obtain

$$p_1^* = 2 + \frac{1}{4}\left(2 + \frac{p_1^*}{4}\right)$$

which has the solution $p_1^* = \frac{8}{3}$. Proceeding we then also obtain $p_2^* = 2 + \frac{(8/3)}{4} = \frac{8}{3}$ which prove both uniqueness and symmetry. Hence in the static price-setting equilibrium we obtain that both firms set the price $p^* = 8/3$. This implies that each firm obtains profits

$$\begin{aligned}
\pi_i^* &= p^*\left(16 - 4p^* + 2p^*\right) \\
&= p^*\left(16 - 2p^*\right) \\
&= \frac{8}{3} \times \left(16 - 2 \times \left(\frac{8}{3}\right)\right) \\
&= \frac{256}{9} \approx 28.44
\end{aligned}$$

(b) When the firms choose the prices to maximize total joint profits

$$\Pi = \pi_1(p_1, p_2) + \pi_2(p_1, p_2) = p_1(16 - 4p_1 + 2p_2) + p_2(16 - 4p_2 + 2p_1)$$

In principle, it could be optimal to set the prices asymmetrically, $p_1 \neq p_2$. However, this is not the case. The first order condition with respect to $p_1$ and $p_2$ are

$$\begin{aligned}
(16 - 4p_1 + 2p_2) - 4p_1 + 2p_2 &= 0 \\
(16 - 4p_2 + 2p_1) - 4p_2 + 2p_1 &= 0
\end{aligned}$$

respectively. From these we see that both equations are linear and, moreover, they are symmetric. Hence the solution will be both unique and symmetric. Setting $p_1 = p_2 = p_M$ in either equation

$$(16 - 4p_M + 2p_M) - 4p_M + 2p_M = 0$$

and solving yields $p_M = 4$.

If we had argued from the outset that a common price was the only possible optimum, then the profit function would have been $\Pi = 2p(16 - 2p)$ and the first order condition for the optimal price $p_M$ would be $2(16 - 2p) - 2 \times 2p = 0$ yielding the same result.

Total joint profits in the collusive agreement are

$$\begin{aligned}
\Pi_M &= 2p_M(16 - 2p_M) \\
&= 2 \times 4 \times (16 - 2 \times 4) \\
&= 2 \times 4 \times 8 \\
&= 64
\end{aligned}$$

with each firm earning half of the joint profits, i.e. $\pi_M = 32$.

(c) Suppose that firm 1 believes that firm 2 will set the collusive price $p_M = 4$. In the optimal deviation for firm 1 choose as price $p_1$ the best response to $p_2 = p_M = 4$. Hence using that we derived the best response function in part (a) above, we have that firm 1's best deviation is to set its price at

$$p^r = p_1(p_M) = 2 + \frac{p_j}{4} = 2 + \frac{4}{4} = 3$$

which, given that $p_2 = p_M = 4$ yield the profits

$$\begin{aligned} \pi^r &= p^r(16 - 4p^r + 2p_M) \\ &= 3 \times (16 - 4 \times 3 + 2 \times 4) \\ &= 3 \times 12 \\ &= 36 \end{aligned}$$

(d) The grim-trigger strategy is as follows: In period $t$, set the monopoly price in $p_M$ if both firms have set the price $p_M$ in all previous period; if, in some previous period, some firm has set a price other $p_M$, then set price $p^*$.

We want to determine when the grim-trigger strategy can sustain collusion at the price $p_M$ as a subgame perfect Nash equilibrium (SPNE). Formally, we need to check when no firm has an incentive to deviate unilaterally along the equilibrium path (the requirement for having a Nash equilibrium.) And we also need to check the same thing off the equilibrium path (the additional requirement for subgame perfection.)

Assume that firm $j$ has adopted the grim-trigger strategy and consider when it is optimal for firm $i$ to do the same. Suppose that we are at time $t$ and both firms have set the collusive price $p_M$ in all previous periods. If firm $i$ also follows the grim-trigger strategy its profits will be $\pi_M$ in every period; hence the total discounted profits for firm $i$ (from the point of view of time $t$) will be

$$\begin{aligned} V_i^M &= \pi_M + \delta\pi_M + \delta^2\pi_M + \ldots \\ &= \sum_{t=0}^{\infty} \pi_M \delta^t \\ &= \pi_M \sum_{t=0}^{\infty} \delta^t \\ &= \frac{\pi_M}{1-\delta} \end{aligned}$$

In contrast, if firm $i$ deviates in period $t$, it will obtain profits $\pi^*$ in all subsequent periods and profits $\pi^r$ in the period of the deviation. Hence the total discounted profits from deviating under the grim-trigger strategy will be

$$\begin{aligned} V_i^r &= \pi^r + \delta\pi^* + \delta^2\pi^* + \delta^3\pi^* + \ldots \\ &= \pi^r + \pi^* \left(\delta + \delta^2 + \delta^3 + \ldots\right) \\ &= \pi^r + \delta\pi^* \left(1 + \delta + \delta^2 + \ldots\right) \\ &= \pi^r + \delta\pi^* \sum_{t=0}^{\infty} \delta^t \\ &= \pi^r + \frac{\delta\pi^*}{1-\delta} \end{aligned}$$

Firm $i$ will have no incentive to deviate precisely when

$$V_i^M \geq V_i^r$$

which is hence equivalent to

$$\frac{\pi_M}{1-\delta} \geq \pi^r + \frac{\delta\pi^*}{1-\delta} \Leftrightarrow$$

$$\pi_M \geq \pi^r (1-\delta) + \delta\pi^* \Leftrightarrow$$

$$\pi_M \geq \pi^r - \delta\pi^r + \delta\pi^* \Leftrightarrow$$

$$\delta (\pi^r - \pi^*) \geq \pi^r - \pi_M \Leftrightarrow$$

$$\delta \geq \delta^{crit} \equiv \frac{\pi^r - \pi_M}{\pi^r - \pi^*}$$

Plugging in the profit levels characterized above we obtain that the critical discount factor is

$$\begin{aligned} \delta^{crit} &\equiv \frac{36-32}{36-\frac{256}{9}} \\ &= \frac{9 \times (36-32)}{9 \times \left(36-\frac{256}{9}\right)} \\ &= \frac{36}{(324-256)} \\ &= \frac{36}{68} \\ &= \frac{9}{17} \\ &\approx 0.53 \end{aligned}$$

Hence the collusive outcome can be sustained through both firms adopting the grim-trigger strategy when $\delta \geq \delta^{crit} = 9/17$.

**(Total 50 marks)**

**END**