

Collusion: Exercises Part 1

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Problem 1 (Collusion in a finitely repeated game)

There are two players, $i = 1, 2$. There are also two time periods, $t = 1$ and $t = 2$. In each period, the following “stage game” is played, where player 1 chooses row and player 2 chooses column,

	A	B	C
A	1, 1	0, 0	5, 0
B	0, 0	3, 3	0, 0
C	0, 5	0, 0	4, 4

That is, the players first play this stage game once. Then, after having observed what the rival did in the first round, they play it a second time, after which the overall game is over. Each player maximizes the discounted sum of all his/her payoffs; the discount factor is denoted δ . Assume that $0 < \delta < 1$.

(a) Consider the one-shot game (i.e., the game you get if the stage game is being played only once). Convince yourself that, in this game, there are two (pure strategy) Nash equilibria: (A, A) and (B, B) . Also convince yourself that:

- The (B, B) equilibrium is preferred by both players to the (A, A) equilibrium.
- Each player would be even better off if they could agree to play (C, C) instead, but this is not a Nash equilibrium of the one-shot game.

(b) Now consider the full game. Under what conditions does there exist a subgame perfect Nash equilibrium in which the players choose (C, C) in the first period? You must prove all your claims. In particular, specify the trigger strategies that the players use. Interpret your results and briefly discuss the key assumptions needed for a equilibrium with collusion to exist in a finitely repeated game.

Problem 2 (Problem 6 in Chapter 10 (page 361) of the book by Church and Ware.)

Suppose that demand is given by $p = A - Q$ and that marginal cost is constant and equal to c , where $A > c$. Suppose that there are n firms and the stage game is Cournot.

(a) Find the critical value of the discount factor to sustain collusion if the firms play a supergame and use grim punishment strategies. Assume that the collusive agreement involves equal sharing of monopoly output and profits

(b) How does the minimum discount factor depend on the number of firms? Why?

Problem 3 (A specific tax in the collusion problem)

Consider the special case of of the previous problem where there are only two firms, $i = 1, 2$. However, suppose that government imposes a specific tax $\tau > 0$ on the product produced by the two firms. This implies that there will be a difference of τ between the price paid by the consumers and the price received by the producers. Argue that the imposition of the tax τ does not affect the sustainability of a collusive agreement since it does not affect the critical discount factor.