Advertising: Exercises

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January 2010

Problem 1

You are the CEO of TOTAL RECALL, Inc., a monopolist producer of facial oil skin-life extender. You need to determine the advertising budget for next year. The marketing department has provided you with three important items of information: (a) The company is expected to sell $10 million worth of the product; (b) it is estimated that a 1% increase in the advertising budget would increase quantity sold by 0.05%; (c) it is estimated that a 1% increase in the product's price would reduce quantity sold by 0.2%.
(a) How much money would you allocate for advertising next year if you applied the Dorfman-Steiner rule?
(b) Suppose the marketing department has revised its estimation regarding the demand price elasticity to a 1% increase in price resulting in a reduction of quantity sold of 0.5%. How much money would you allocate to advertising after getting the revised estimate? Why has the optimal advertising expenditure gone down?

Solution Problem 1

(a) Recall the Dorfman-Steiner rule,

$$\frac{\varepsilon_A}{-\varepsilon_P} = \frac{A}{pQ}$$

where ε_A is the elasticity of demand with respect to advertising expenditure, ε_P is the elasticity of demand with respect to price, A is optimal advertising expenditures, p is price and Q is demanded quantity. Rearranging yields

$$A = R\frac{\varepsilon_A}{|\varepsilon_P|}$$

where $R = pQ$ is revenue. Plugging in the values given we obtain that the optimal advertising budget would be

$$A^* = 10\frac{0.05}{0.2} = 2.5$$

million dollars.

(b) Simply replacing in the formula yields

$$A^* = 10\frac{0.05}{0.5} = 1$$

million dollars. The greater price sensitivity of demand has made advertising less profitable.

Problem 2 (Problem 13.7 from Cabral)

Your company sells expensive, branded fountain pens. There are 100,000 people aware of your pens. Each of these 100,000 peoples has his or her own willingness to pay for your pens. These willingness-to-pay numbers are uniformly distributed between \$0 and \$500. So your demand curve is given by $Q = 100,000\,(1 - p/500)$. Your marginal cost per pen is \$100. Well-versed in economics, you are pricing your pens at \$300 each, and selling 40,000 pens, generating a profit of \$8 million.

You have just become brand manager for these fountain pens. The previous brand manager engaged very little in advertising, but you are considering running a major promotional campaign to build your brand image and visibility. You are considering two possible advertising campaigns; call them "Build Value" and "Expand Reach". You will run either one of these campaigns or none at all; you cannot run both.

The "Build Value" campaign will not reach any new potential customers, but it will increase the willingness-to-pay of each of your existing 100,000 customers by 25%. This campaign costs \$2.5 million to run.

The "Expand Reach" campaign will expand the set of potential customers by 25%, from 100,000 to 125,000. The 25,000 new customers reached will have the same distribution of willingness-to-pay as the preexisting 100,000 potential customers (namely, uniformly distributed between \$0 and \$500). This campaign costs \$1.8 million to run.

(a) If your choice is between running the "Build Value" campaign and running no campaign at all, would you choose to run the "Build Value" campaign? Show your calculations.

(b) If your choice is between running the "Expand Reach" campaign and running no campaign at all, would you choose to run the "Build Value" campaign? Show your calculations.

(c) What choice would you make in this situation: Run the "Build Value" campaign, run the "Expand Reach" campaign, or run neither?

Solution Problem 2

(a) Consider the demand after the BV campaign. Prior to the campaign, the $N_0 = 100,000$ consumers had a willingness to pay that was uniformly distributed between 0 and 500. The BV campaign boosts the willingness to pay (WTP) for each of the N_0 consumers by 25 percent. Hence a consumer with an initial WTP of 0 will still have a WTP of zero; a consumer with an initial WTP of 100 now has a WTP of 125, and a consumer with an initial WTP of 500 now has a WTP of 625. Hence WTP is now uniformly distributed between 0 and 625. This implies that demand is given by

$$Q^{BV}(p) = N_0 \times \left(1 - \frac{p}{625}\right)$$

or, equivalently, an inverse demand curve of

$$p^{BV}(Q) = 625\left(\frac{N_0 - Q}{N_0}\right).$$

Profit maximization after the BV campaign involves choosing Q so as to maximize

$$\begin{aligned}
\pi^{BV}_{gross} &= Q \times p^{BV}(Q) - cQ \\
&= Q \times \left(p^{BV}(Q) - c\right) \\
&= Q \times \left(625\left(\frac{N_0 - Q}{N_0}\right) - 100\right) \\
&= 525Q - 625\frac{Q^2}{N_0}
\end{aligned}$$

The first order condition is

$$525 = \frac{1250Q}{N_0}$$

or

$$Q^{BV} = \frac{525 \times N_0}{1250} = \frac{525 \times 100000}{1250} = 42\,000.$$

The corresponding price is

$$\begin{aligned}
p^{BV} &= p^{BV}\left(Q^{BV}\right) \\
&= 625\left(\frac{N_0 - Q^{BV}}{N_0}\right) \\
&= 625\left(1 - \frac{42}{100}\right) \\
&= \frac{725}{2} \\
&= 362.5
\end{aligned}$$

Optimal profits after the BV campaign (net of advertising costs) are then

$$Q^{BV}\left(p^{BV} - c\right) = 42\,000 \times (362.5 - 100) = 11,025,000.$$

Hence, subtracting the cost of the campaign, profits after the BV campaign are

$$\begin{aligned}
\pi^{BV}_{net} &= 11,025,000 - 2,500,000 \\
&= 8,525,000
\end{aligned}$$

which is larger than the initial profit of 8 million.

(b) Consider now the "Expand Reach" (ER) campaign. This campaign will leave the distribution of WTP as uniform between 0 and 500. However, it will increase the number of potential customers by 25 percent from $N_0 = 100,000$ to $N_1 = 125,000$. After the ER campaign, the demand is given by

$$Q^{ER}(p) = N_1 \times \left(1 - \frac{p}{500}\right)$$

or, equivalently, an inverse demand curve of

$$p^{ER}(Q) = 500\left(\frac{N_1 - Q}{N_1}\right).$$

Profit maximization after the BV campaign involves choosing Q so as to maximize

$$\begin{aligned}
\pi^{ER}_{gross} &= Q \times p^{ER}(Q) - cQ \\
&= Q \times \left(p^{ER}(Q) - c\right) \\
&= Q \times \left(500\left(\frac{N_1 - Q}{N_1}\right) - 100\right) \\
&= 400Q - 500\frac{Q^2}{N_1}
\end{aligned}$$

The first order condition is

$$400 = \frac{1000Q}{N_1}$$

or

$$Q^{ER} = \frac{400 \times N_1}{1000} = \frac{400 \times 125000}{1000} = 50,000.$$

The corresponding price is

$$\begin{aligned} p^{ER} &= p^{ER}\left(Q^{ER}\right) \\ &= 500\left(\frac{N_1 - Q^{ER}}{N_1}\right) \\ &= 500\left(1 - \frac{50}{125}\right) \\ &= 300 \end{aligned}$$

Optimal profits after the ER campaign (net of advertising costs) are then

$$Q^{ER}\left(p^{ER} - c\right) = 50\,000 \times (300 - 100) = 10\,000\,000$$

Hence, subtracting the cost of the campaign, profits after the ER campaign are

$$\begin{aligned} \pi^{ER}_{net} &= 10,000,000 - 1,800,000 \\ &= 8,200,000 \end{aligned}$$

which is larger than the initial profit of 8 million. Hence running ER would increase profits.

(c) The "Build Value" campaign adds $525,000 to profits while the "Expand Reach" campaign only adds $200,000. So you should run the "Build Value" campaign.

Bonus Problem: Signalling Games

Englefield Green, 1852. The last duel in England.

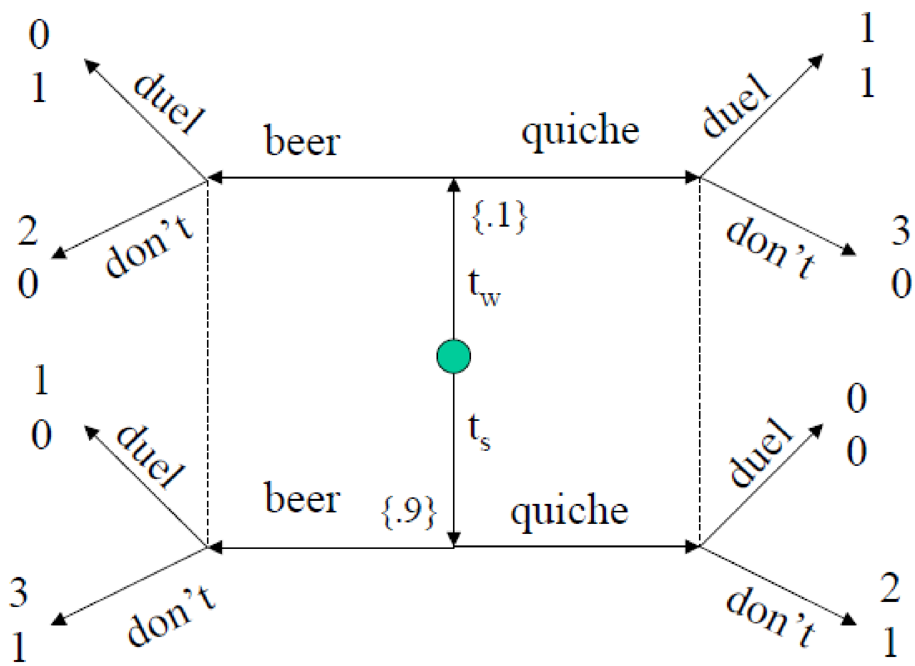
It was between two French refugees, Lt. Frederic Constant Cournet and Emmanuel Barthelemy. Cournet was supposed to have been the better prepared for a sword duel. Barthelemy, an extremely questionable individual (responsible for at least two murders by 1852), manipulated Cournet into challenging him (supposedly over comments Cournet made about Barthelemy's girlfriend), and chose pistols for the weapon. He killed Cournet, *and was subsequently arrested for murder. However Barthelemy managed to convince the jury that it was not a homicide as in the normal sense of the word, and was acquitted.*

Suppose Cournet can observe what Barthelemy had for breakfast but cannot observe if Barthelemy is strong or weak.

Definitions:

A pooling equilibrium is an equilibrium in which all types of sender send the same message.

A separating equilibrium is an equilibrium in which all types of sender send different messages.

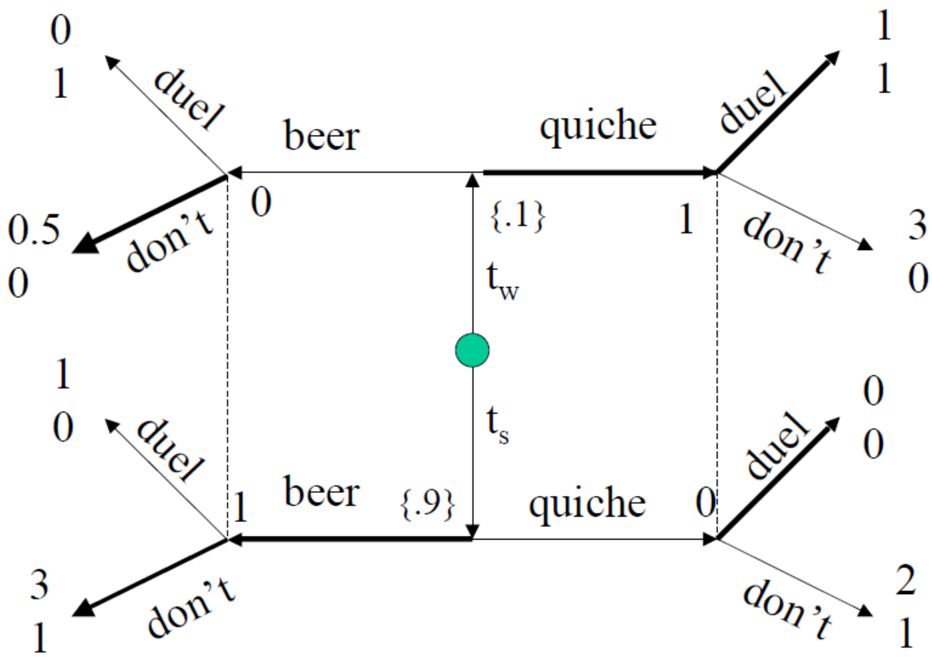


Is there a separating (informative) equilibrium in this game? What about in the next game?

Should Cournet choose to fight Barthelemy in this equilibrium after observing him have beer for breakfast?

Solution:

The main idea when solving a Bayesian game (a game where you don't know your opponent's type) is that you need to have beliefs about the other player's type and your



beliefs need to be consistent with the equilibrium. For example, if only strong types have beer, then whenever you see someone drinking beer you know he is a strong type. For more details on Bayesian Games see Lecture 7 from my Game Theory class or read M.Osborne, Chapters 9, 3.5.

The first game has no separating equilibrium.

Consider strategies for player 1 of the type (X,Y)=(play X if weak; Y if strong) and for player 2 (X,Y)=(play X if you see beer; Y if you see quiche)

There can be two candidate strategies for player 1 to have separation, (B,Q) or (Q,B).

Start with (Q,B)

Then the best response of player 2 is (No duel, Duel)

But then, when player 1 is weak he should have Beer instead of quiche and get a payoff of 2>1.

Can (B,Q) be part of a separating equilibrium?

Then the best response of player 2 is (Duel, No Duel)

But then, when player 1 is weak he should definitely have quiche instead of beer and get a payoff of 3>0, so this can't be an eq.

Consider the second game.

Start with (Q,B)

Then the best response of player 2 is (No duel, Duel)

When player 1 is weak he gets a payoff of 1 when following the equilibrium and only 0.5 in case he deviates.

When player 1 is strong he gets a payoff of 3 when following the equilibrium and only 0 in case he deviates.

So, no player wants to deviate => this is an equilibrium.

Can (B,Q) be part of a separating equilibrium?

Then the best response of player 2 is (Duel, No Duel)

But then, when player 1 is weak he should definitely have quiche instead of beer and get a payoff of 3>0, so this can't be an eq.

What does this all mean? Signalling games apply to many fields, among others to college education or advertising. Separating equilibria do not always exist. Depending on the parameteres of the game it can be that you can tell a company's quality from the ads it makes or it can be that all companies make the same sort of ads so there is no informative signal.