Information Revealing Speculation in Financial Markets and Company Takeovers

Sotiris Georganas* Michael Zähringer†

November 2008

Abstract

We investigate the role of an initial public offering (IPO) in maximizing the original owner's revenue. The value of a company consists of two components: future cash flows and a private benefit associated with control which only the majority owner enjoys. By issuing a fraction of the asset in a financial market (partial IPO) with a large number of small liquidity constrained shareholders, the seller elicits information in order to extract additional rents from the potential buyer of the controlling share of the company. We compare this two-step procedure to going public with the entire company, and with an auction with optimal reserve price. We find conditions, under which the two-step procedure outperforms the other two mechanisms. Our model provides a possible explanation for an IPO followed by a take-over as it is frequently observed in practice. Additionally, it provides a framework for the discussion of stockmarket regulations on takeovers and minority shareholder protection. Minority rules do not only affect the efficiency of takeovers which is the usual focus of the analysis, but also have a significant effect on the information content of market prices.

JEL Classification: G14, D82, D84 Keywords: Initial Public Offering, Information Revelation, Take over

*Ohio State University, email: georganas.1@osu.edu
†Universitaet Bonn, email: michael.zaehringer@uni-bonn.de

1 Introduction

The owner of a highly valuable and divisible asset contemplating to sell it, is faced traditionally with two options: an auction with a reserve price and an initial public offering (IPO). The typical setting of an IPO is a large market where small players have pieces of relevant information. The information aggregation properties of these markets are the main points of interest. On the other hand, an auction is used to sell an asset to relatively few, big players, who act strategically. Revenue is maximized by manipulating the incentives of the buyers to reveal their true valuations. While the seller is imperfectly informed, the buyers' valuations are not necessarily private information. There may exist third parties with relevant information and inducing them to share it is of great interest to the seller, as it influences her ability to extract surplus from a transaction.

In this paper we analyze an alternative two-stage mechanism which brings forth a higher revenue by revealing the information of the third parties and allowing the seller to appropriate more rents from the buyers. It consists of

1. an emission of a minority part of the shares (partial IPO)

2. an auction with optimal reserve price for the rest of the shares.

The two stage mechanism is a mixture of the one stage alternatives and combines their respective advantages. It exploits the presence of the informed agents to gather information about the buyers' valuations and uses this information to set an optimal reserve price in the second stage.

Both one-stage options mentioned above, are often encountered empirically. Russia for example has privatized many companies (Yukos, Sibneft) by direct bargaining with the prospective owners. Germany has organized large auctions for former East German assets (through the Treuhandanstalt). On the other hand, the Japanese monopolistic power utility was recently privatized through an IPO for all shares. But there are cases, where due to a variety of reasons, first a part of the company is sold through an IPO and then the majority is sold to a strategic partner. Such a two-stage method is widely followed in Europe[1] in countries like France (EdF), Greece (Hellenic Telecom, Emporiki Bank), the Scandinavian countries

[1] One can plausibly explain the use of such methods by the need for public revenues without the backlash privatizations tend to bring in these countries. However we do not think this method would be continuously used if it brought consistently suboptimal results.

or during the privatization programmes in Central Europe (e.g. Czech Telecom). Gaz de France is another prominent example. After an initial offer of ca. 20% of its shares to the public, is it now being effectively privatized through bargaining with Suez. And lastly there are cases where the public unloads its shares in a series of public offerings, as in the case of Deutsche Telekom, TeliaSonera, National Bank of Greece and others. Thus the mechanisms we examine represent options encountered in real markets. We will argue that the two stage mechanism is the most effective one under some conditions, based on three basic features: the existence of big buyers with control benefits, the existence of small agents with information about these and minority shareholder protection rules in financial markets. We will also argue that the same forces that make our mechanism effective are at play whenever a listed company is considered a possible takeover target and thus influence the market price of shares.

Small informed agents are ubiquitous in financial markets. These can be investment banks, pension funds or even individual analysts, who acquire this information in the course of their everyday business. Before we proceed to a further analysis of their information, it will be of use to dissect the value of the company in two parts: a cash flow part and a corporate control part (see for example Zingales, 1995). Cash flow rights are enjoyed by all shareholders, in proportion to their equity stake. Therefore we assume the cash flow part is the same for every shareholder, and commonly known. The corporate control part however, depends on who controls the management of the company. Every possible owner of the company has different benefits she can derive from controlling the company, which are known only to herself. These benefits accrue only to the owner and can range from the purely psychological value of being in control (Aghion, Bolton 1992) to perks enjoyed by top executives[2]. An additional reason for private control benefits is that the ownership of some share might affect other shareholdings of an individual or company. For example Porsche recently acquired a 20% stake in fellow carmaker Volkswagen. This control gives it strategic benefits that the other shareholders of Volkswagen do not enjoy.

Control benefits can be quite large.[3] There are empirical studies estimating them, based

[2]Perks can be the use of corporate assets and infrastructure, club memberships, special discounts etc but also importantly other indirect benefits. For example, the suitability if a new subcontractor or partner in a new project is not always clear. The person who has the power to choose a partner can expect personal benefits from this choice, without any anticipated damage to, or reaction from, the shareholders.

[3]A spectacular example was observed in the recent takeover of TXU, where KKR and Texas Pacific offered a 25% premium over the average closing price in the 20 days before the offer. The New York Times actually reported the control benefits must be even higher, as the markets (some informed agents?) responded by

mainly on the different prices paid for individual shares and for packages of shares carrying the control of the company. The size of this discrepancy is found to be quite significant, for example by Dyck and Zingales (2004) it is estimated at 14% on average. Thus the control benefits are a private value and constitute a sizeable portion of the possible total value of the company to any particular majority owner.

The informed agents have some knowledge of these private values, but no control benefit value of their own. This is due to the fact that the banks and analysts we mentioned above do not have the intention or the capacity to manage the company. They are just buying shares with the speculative motive to resell them at a higher price. Usually these financial investors are liquidity constrained[4] and thus unable to influence the outcome of an eventual sale of 100% of the company through e.g. an auction. Under some circumstances it can be of benefit to the seller if these financial investors somehow revealed their information or participated in the sale. However, the identity of the informed financial investors is unknown, so the use of a direct mechanism to elicit their information is impossible. And a simple auction is not a solution either, as the small financial investors cannot possibly influence the outcome, due to their liquidity constraints. A big impersonal market, e.g. the stock market, is a natural alternative to these mechanisms.

In most important financial markets, small investors are protected by minority shareholder protection regulation, in particular by a *sell out rule*. This rule states that when any investor buys more than a certain percentage of the shares of a company (ranging from 30% to 50%) she has to offer a *fair price*[5] for the shares of all other remaining minority shareholders. Such regulation has very important consequences, as it allows the small investors to acquire stakes in the company using their information regarding its value to a potential buyer, without fearing they will be bypassed in the takeover agreement. The presence of these speculating investors, who just buy to resell, could be a factor raising the revenue of sellers conducting IPOs.[6]

raising the stock price even higher than the offered by KKR.

[4]Most financial investors are small relative to the size of the companies they study. In the case of large investment banks their size is not that insignificant, but the investors who have the relevant information will belong to a division of this bank, which surely can not use all resources of the company. Additionally, regular banks almost never buy majority stakes in a whole corporation from a non-related field. This can be due to regulation and/or diversification reasons.

[5]According to a recent EU directive, a fair offer is an offer equal to the maximum price the acquiring investor has paid for shares of the company under sale, in the recent months. The length of the period considered is allowed to vary in the member states between 6 and 12 months.

[6]From this hypothesis it follows we should expect these investors to be more active, the higher the

Our model examines the role of these speculators and provides insights in the way minority protection influences the investors' behavior and how it enhances the information aggregation properties of financial markets. We claim this is an unintended and not much studied effect of minority shareholder protection. The usual analysis deals with this protection on the basis of its effects on the efficiency of takeovers (see the seminal paper of Grossman and Hart, 1980) or perceives it as a rule to protect small shareholders from exploitation by the private benefit seeking majority owners. In this paper we show how, further to these effects, minority protection makes markets informationally more efficient, which can benefit all types of investors, be they in the minority or majority. Our model also applies to cases where an agent attempts a takeover of an already listed company. Our results can thus be used to answer questions regarding the reaction of the share price and its information content after the announcement of the takeover attempt. We find that under some conditions the target company can plausibly claim that its share is undervalued, even after the takeover is announced[7].

Our model differs from most IPO papers in the techniques applied, as our focus is on the special informational structure outlined above and how a strategic player (the seller) can use it to his advantage. We want to abstract from other phenomena like the strategic behavior of the underwriting banks and consequent underpricing which are often discussed in this literature on the role and design of IPOs when outsiders can generate information about the firm (see Rock 1986, Benveniste and Spindt 1989). Due to this, we build mainly upon the theoretical literature on financial markets, among others the seminal paper of Grossman and Stiglitz (1980). They use a simple model, where agents can either be fully informed or uninformed. Assuming traders have CARA utility function and that the return of the asset is normally distributed, they are able to find linear equilibria. The authors proceed to analyze how information is conveyed from the informed to the uninformed through the price.[8] We additionally include the standard assumption of noise traders (see for example Hellwig 1980) who bring a stochastic element to the models and allows an only partial information revelation in the markets. The assumptions of CARA utility and normally distributed random values, are crucial in these and most other papers in the literature (e.g. Verrechia 1982, Admati

possibility of an eventual takeover after the IPO. Actually, empirical evidence suggests that many IPOs are followed by an eventual merger or acquisition by another company. Pagano, Panetta and Zingales (1998) find that IPOs are followed by a much higher turnover of control than that of similar privately held companies.

[7] Yahoo has recently claimed its share was undervalued, while bargaining over a merger with Microsoft.

[8] In our model information flows from the informed traders to the uninformed seller.

1985) for the existence of a tractable model with a linear solution. Unfortunately, as we shall see, in our two-stage mechanism it is guaranteed that the posteriors will not be normally distributed, which precludes the use of standard techniques. Due to this, we follow Barlevy, Veronesi (2000) which is one of the few tractable models which do not make use of these assumptions. The authors construct a model with a binomial state space and risk neutral informed/uninformed traders.

There are few theoretical models asking similar questions to our paper. Boone and Goeree (2005) explore the sale of an asset when there is a single insider bidder who possesses better information about the asset's risky value and bidders differ in their costs of exploiting the asset. The insider's presence results in a strong winner's curse for the uninformed bidders and devastates expected revenue. The authors show that the optimal mechanism discriminates against the informationally advantaged bidder to ensure truthful information revelation by employing a two stage mechanism. In the qualifying auction, non-binding bids are submitted to determine who enters the second stage, which consists of a standard optimal auction (i.e. second-price auction with an optimal reserve price).

Zingales (1995) focuses on the role of an IPO when there is perfect information about the buyer's impact on cash flow and the control premium. He shows that direct bargaining maximizes the proceeds from the sale of the control right. On the other hand, an IPO is more appropriate to extract rents from cash-flow rights to dispersed shareholders. The decision whether to go public and which fraction to issue depends on the trade off between the two effects. Biais et al (2002) discuss optimal IPO mechanisms when there exist professional investors with private information and liquidity constrained retail investors. However private control benefits and a possible takeover of the company are not considered in this paper. Thus, there is no role for speculating small agents who are covered by minority shareholder protection, which is crucial in our model. In Subrahmanyam and Titman (1999) firms do IPOs because the price revealed in secondary market trading can be useful. This paper shares with our model the market microstructure approach to how information gets reflected in the firm's price. However the analysis focuses on the way that information in the stock market can help enterpreneurs make better production choices. A possible sale of the company and agents' information about the values of potential buyes is not considered.

Section 2 introduces the model and presents the main results followed by an illustrative numerical example including comparative statics in section 3. Remarks and extensions are presented in section 4. Section 5 concludes. Omitted proofs can be found in the appendix.

2 The model

There are two assets. One is a riskless asset, with return R, scaled without loss of generality to zero. The other asset is a firm with a total value θ, which is the sum of the common value created by the cash flow part plus the private control benefit, which can differ depending on who owns the company. To simplify the setup, we assume there is only one *strategic investor* B interested in acquiring the company and her control benefits are binomially distributed as in Barlevy and Veronesi (2000). We further assume the cash flow part is equal to zero.[9] This gives us following distribution for the total value θ:

$$\widetilde{\theta} = \begin{cases} \overline{\theta} & \text{with prob} \quad \sigma \\ \underline{\theta} & \text{with prob} \quad 1-\sigma \end{cases} .$$

From the discussion of the control benefits, it follows we can assume they are always positive. We have $0 < \underline{\theta} < \overline{\theta}$.

The prior probability σ of θ being high is assumed to be low:

$$0 < \sigma < \underline{\theta}/\overline{\theta} \tag{A.1}$$

As we shall see later, this assumption means that without additional information the optimal take-it-or-leave-it (tioli) offer to a single buyer is $\underline{\theta}$.

There is a continuum of financial investors $i \in [0,1]$ whose valuation of the firm is zero. These agents all have the same endowment of money, which we set equal to 1 and we assume that their individual endowment is smaller than the lowest possible value of the company, $\underline{\theta} > 1$.[10] We also assume the financial investors are risk neutral, so they invest all their endowment in the asset with the highest expected return. This allows us to avoid the usual problems of investors having a nonlinear demand, as described in the introduction. Additionally, we assume that the financial investors are liquidity constrained and short selling is prohibited.

[9] Any value of the cash flow part, as long as it is deterministic, results in a binomial distribution of the total value.

[10] An additional technical assumption is useful here. We assume that the (sum of the) endowments of the financial investors is greater than half the maximum value of the company $1 > 0.5\overline{\theta}$. This is needed so that it is possible for all financial investors together to buy half the company for any price up to the maximum value of the company. Thus, in the case of a partial IPO they can buy all offered shares at a fully revealing price. Note, this assumption is not very restrictive, but just a result of the investors being distributed over the unit interval. The unit interval makes the analysis simpler but also makes their total endowment equal to their individual endowment. Alternatively we would assume that investors are distributed in the interval $[0,x]$ and that their total endowment is large enough, $x > 0.5\overline{\theta}$.

This precludes spending more than their endowment. These assumptions represent the idea that there are many small financial investors, with no market power. All these investors are assumed to be informed[11] of the true value of θ.

The original owner chooses a mass of shares λ of the company to sell in an IPO and $1-\lambda$ to offer subsequently to the strategic investor. He enjoys no private control benefits[12], thus it is always efficient to sell the control of the company to the strategic investor. In the setup we have described so far, the revenue maximization problem of the seller can be quite trivial. Given that the total wealth of the investors is high enough to clear the market, she can offer any mass of shares (though less than 0.5 to avoid ceding management control) through an IPO in the first stage, announcing she will use the price of the IPO as a reserve price for the rest of the shares in the second stage. For all prices less than θ, aggregate demand will exceed the supply as informed investors buy all shares up to a price equal to θ. Note that we have as many possible realizations of the market clearing price as states of the world, in this case two. This invertible price function leads to full information revelation. The seller uses this information to extract all rents in the second stage, by charging the strategic investor her full value to transfer control. The informed, financial investors, subsequently sell all their shares to the strategic investor at a price equal to θ, due to the fair price rule described in the introduction.

Of course this example is highly stylized. Complete information revelation is very rare in a financial market. A common reason is that in virtually all markets there exist some so called *noise traders,* agents who rationally ignore the fundamentals and trade for reasons exogenous to this market (e.g. liquiditiy needs, for a nice discussion see Shleifer and Summers 1990) or traders with insufficient experience, bounded rationality etc Such traders will usually hinder full information revelation. It is important to investigate if the mechanism we propose is robust to this almost ubiquitous feature of real markets. Thus, according to standard practice we introduce some noise into the system, whic precludes prices from revealing all available information.

[11] We will later explain what happens when this is not true and how the key insights of our model transfer to the more general case, where we allow for the presence of uninformed, rational players. Also, the assumption that speculators are perfectly informed can be replaced by the assumption that every speculator gets a noisy signal, with the noise having a zero mean and cancelling out on aggregate. It is straightforward to extend the results of our model to this case, however the analysis would be unnecessarily complicated.

[12] Actually he might have control benefits which we suppose are lower than the potential buyer's. Especially when talking about privatizations, we could speak about the higher efficiency private ownership brings.

The noise traders possess total wealth $w > 0$.[13] Due to exogenous reasons which will not be motivated strategically in the current study they spend a random share $\widetilde{x}$ of their wealth buying stock.[14] Let p denote the price of the total asset; total noise trade becomes

$$x_0(p) = \widetilde{x}\ \frac{w}{p} \tag{A.2}$$

Since this model is describing an IPO, we do not allow the noise trade to become negative, i.e. there can be no short sales.

As is usually found in the literature, the seller cannot distinguish between demand coming from the noise traders or from the informed investors, so she is not able to invert the price function to reveal the state of the world. We furthermore assume that w is large enough to keep the market liquid for a given part of the shares λ that are offered and for any reasonable price below the maximum value of θ,

$$w > \frac{1}{2}\overline{\theta} \tag{A.3}$$

The game proceeds as a sequence of seven steps.

Step 1: Random draw of θ out of a binomial distribution with $prob(\theta = \overline{\theta}) = \sigma$.

Step 2: Choice of λ

The seller S selects a portion $\lambda \in [0, \frac{1}{2}]$ of the company to be sold through an initial public offer. The fraction λ is publicly announced. In the case $\lambda = 0$ steps 3 to 5 are omitted.

Step 3: Random draw of the noise trader wealth investment share $\widetilde{x} \in (0, 1]$. The share $\widetilde{x}$ has a twice continuously differentiable and logarithmically concave density f, which is positive on the whole interval $[0, 1]$. No information about the realization of $\widetilde{x}$ is given to any player.

Step 4: Given a θ, each investor $i \in [0, 1]$ chooses a piecewise continuous demand schedule $x_i\,(p)$. This schedule assigns a set of demands $x_i(p)$ to every $p \geqslant 0$, with $\sup\{x_i(p)\}p \leqslant 1$ (due to the liquidity constraint). No other player than i receives any information about $x_i\,(p)$.

Step 5: Market for the stock of the initial public offer

[13] Recall that the endowment of the informed investors is equal to one. This is not restrictive, as for our results only the relative size of their endowment to w matters.

[14] To allow for prices below $\underline{\theta}$ we need an additional assumption that informed traders are relatively poor, i. e. $w_I = 1 < \lambda\underline{\theta}$

The price p of the stock is determined using the equilibrium in demand functions concept (Grossman 1981, for an application related to ours see Kyle 1989) as follows. Define the aggregate demand of the informed investors

$$x_I(p) = \int_0^1 x_i\,(p)\,di$$

for values of p such that the integral on the right hand side exists[15].

If the market equation

$$\frac{\widetilde{x}\ w}{p} + x_I(p) = \lambda$$

has a smallest solution p_o for p, then p_o is the publicly announced market price. We speak of market failure[16] if no smallest solution p_o exists. In this case the price is set at $+\infty$ and no shares are sold in the IPO.

Step 6:

The seller makes a take it or leave it offer $r \geqslant 0$ to the buyer. This means that S is willing to sell the fraction $1 - \lambda$ of the company for $r(1 - \lambda)$ money units to B. The offer r is then made public.

Step 7:

The buyer can accept ($\psi = 1$) or reject ($\psi = 0$) the offer r of the seller. The game ends with step 7, unless it has already ended in step 5.

2.1 Equilibrium

We focus on pure strategies. A strategy combination will always be a combination of pure strategies and an equilibrium will be an equilibrium in pure strategies. A strategy of a player is defined as a function which assigns a choice at every information set u of the player.

[15] Note the demand schedule is a correspondence, as we allow the investors to be indifferent between many demands for a given price. We use here the integral of a correspondence, for a definition see Handbook of Mathematical Economics, p. 206.

[16] There are three possible reasons for a market failure. (a) The integral defining f(p) may not exist for any price. (b) The market equation has no solution. (c) The market equation has no smallest solution (the set of all solutions is an open interval). In real markets such market failure can happen, if for example the computerized systems overload or the software is confronted with unforeseen contingencies. We specify that in the case of market failure no orders are executed. Note that in our equilibrium there will never be a market failure.

Information sets Player S has one information set u_2 at step 2 and an information set $u_6(\lambda, p_0)$ for every pair (λ, p_0) with $\lambda \in (0, \frac{1}{2})$ and $p_0 > 0$ at step 6.

An investor i has an information set $u_i(\lambda, \theta)$ for every pair (λ, θ) with $\lambda \in (0, \frac{1}{2})$ and $\theta \in \{\underline{\theta}, \overline{\theta}\}$.

Player B has one information set $u_7(\lambda, p_0, r)$ for every triple (λ, p_0, r) with $\lambda \in (0, \frac{1}{2}]$, $p_0 > 0$ and $r \geqslant 0$ at step 7. Player B also has an information set $u_7(0, r)$ for $\lambda = 0$ and every $r \geqslant 0$.

Strategies A strategy φ_S of S assigns a $\lambda \in [0, \frac{1}{2}]$ to u_2 and an offer $r(\lambda, p_0) \geqslant 0$ to every u_6.

A strategy φ_i of an investor i assigns a demand schedule $x_i(\theta, p, \lambda) = \varphi_i(u_i(\lambda))$ to every one of his information sets $u_i(\lambda)$. This schedule must have the properties mentioned in the description of step 4.

A strategy φ_B of B assigns $\varphi_B(u_7) \in \{0, 1\}$ to every information set $u_7(\lambda, p_0, r)$ or $u_7(0, r)$ of player B.

A strategy combination φ is a collection of exactly one strategy φ_S for S, an φ_B for B as well as exactly one strategy for every investor $i \in [0, 1]$. A strategy combination is symmetric if for every λ and θ all investors i gave the same demand schedule $x_i(\theta, p, \lambda) = \varphi_i(\lambda)$. A combination φ' is a deviation from φ, if the strategy of exactly one player is different in φ and φ'. This player is called the deviator from φ in φ'. A strategy combination is an equilibrium if no deviation φ' from φ yields a higher payoff to the deviator.

Table 1 shows the payoffs. Payoffs are calculated by assuming that in case of market failure or rejection of the reserve price, the company is liquidated. Then the payoff of the seller S and the buyer B is zero.

			$\mathbf{A}_S$	$\mathbf{A}_i$	$\mathbf{A}_B$
$\lambda \in (0, \frac{1}{2})$	market price p_0	$\psi = 1$	$\lambda p_0 + (1-\lambda) r$	$(r - p_0) x_i(p_0)$	$\theta - r$
	market price p_0	$\psi = 0$	0	$-p_0 x_i(p_0)$	0
$\lambda = 0$ or market failure	$\psi = 1$		r	0	$\theta - r$
	$\psi = 0$		0	0	0

Table 1: Player payoffs. A_S is the payoff of the seller, A_B is the payoff of the buyer, A_i is the payoff of the informed investors.

In the following we focus on the substructure of the game where some $\lambda \in (0, \frac{1}{2})$ has already been chosen[17], that is we treat λ as exogenous and the strategies of the buyer B and the investors i do not depend on λ[18]. We then solve by backward induction. The seller knows the equilibrium strategies of the players in the first stage, this means he knows the (stochastic) equilibrium relationship of the price with the unknown variable θ and thus can build a price rule. He uses it in the second stage to determine his posterior beliefs about the value of the asset, after observing the stock price. We then characterize the optimal take-it-or-leave-it (tioli) offer the seller will make to the strategic investor, focusing on subgame perfect Nash equilibria where the seller is allowed to use a cutoff rule. Using the outcome of the second stage to calculate the returns for the buyers in the first stage, we derive the optimal demand schedules.

2.2 Second stage

In the second stage the seller knows the price rule, which she uses to update her beliefs about the state of the world. Given the posterior probability $\widehat{\sigma}$[19] (which is a function of the observed price p) she offers the asset to the investor for the price r. If the investor rejects her offer, the asset will be liquidated which results in zero payoffs for all parties. The seller's expected revenue v, given she observes p and offers r, is:

$$E[\widetilde{v}|\, p, r] = \begin{cases} \widehat{\sigma}(p) \cdot \overline{\theta} & \text{if} \quad r > \underline{\theta} \\ \underline{\theta} & \text{if} \quad r \leq \underline{\theta} \end{cases}$$

Revenue equals $\underline{\theta}$ with certainty if the seller asks for a resserve lower than $\underline{\theta}$ and if the reserve is higher than that it is only accepted when θ is high, which is true with probability $\widehat{\sigma}(p)$. It is obvious that all reserve prices other than one of the two realizations of theta are dominated.[20] If the seller charges more than $\overline{\theta}$ the company is liquidated and her payoff is zero. This offer is dominated by $r = \overline{\theta}$ which results in positive revenue if the buyer's control premium is high. The converse holds for reserve prices below $\underline{\theta}$ that are always accepted. A transaction price between the two realizations only occurs if $\widetilde{\theta} = \overline{\theta}$ and is therefore dominated

[17] For an actual calculation of an optimal λ we refer to the numerical example.
[18] This is true because the best responses of B and the investors i are the same for all $\lambda \in (0, \frac{1}{2})$.
[19] We explicitely determine the seller's posterior beliefs in the next subsection.
[20] The simple optimization problem the seller faces in the second stage is one major advantage of our binary setup, as it allows for tractability of the model. In most other cases reserve prices can only be defined implicitly.

by $\overline{\theta}$. Thus, in equilibrium the seller offers

$$r^*(p) = \begin{cases} \overline{\theta} & \text{if} \quad \widehat{\sigma}(p) > \underline{\theta}/\overline{\theta} \\ \underline{\theta} & \text{if} \quad \widehat{\sigma}(p) \leq \underline{\theta}/\overline{\theta} \end{cases} \tag{1}$$

and the buyer accepts whenever the reserve price r^* does not exceed his value θ.

The seller's second-stage behavior feeds back to the valuation of financial investors. Thus, to determine the first stage outcome, we have to make a conjecture about the optimal reserve price which in turn depends on the IPO outcome itself. By assumption, only aggregate demand can be observed. Relevant information can therefore solely be revealed by the market-clearing price. Suppose that the price rises weakly monotonically in the true value θ, as we shall show to be true in equilibrium. This monotonicity means that a high IPO price signals a high value of the company. Then, due to the binary state space it seems sensible that the seller's offer will have a single discrete jump in the IPO price. Thus we focus on strategies where the seller uses the following cut-off rule p^* in the price interval $(\underline{\theta}, \overline{\theta})$:

$$r^* = \begin{cases} \overline{\theta} & \text{if} \quad p \in (p^*, \overline{\theta}] \\ \underline{\theta} & \text{if} \quad p \in [\underline{\theta}, p^*] \end{cases} \tag{2}$$

If the financial investors anticipate this cut-off rule correctly, their demand has to be zero for any price $p \in (\underline{\theta}, p^*]$, independently of their information, since the price will then exceed the proceeds from the second stage ($\underline{\theta}$ according to (2)). On the other hand, if the price lies in the interval $(p^*, \overline{\theta}]$ investors' demand does depend on their information. If the actual value and the reserve price (resulting from the IPO price) coincide, they invest all their wealth in the risky asset. In the contrary case however, their demand is zero.

Let us now consider prices outside the interval $(\underline{\theta}, \overline{\theta})$. Financial investors act rationally, i.e. they never buy stock for a price above $\overline{\theta}$. For prices below $\underline{\theta}$ we have to proceed one step further: the seller could now potentially choose a high reserve price, which would give the informed agents a zero payoff in case the company value is in fact low. In such a case the investors' demand schedule x_I should become zero, even for very low IPO prices. On the other hand, if the reserve price is indeed low, the informed agents demand becomes positive for every price lower than $\underline{\theta}$. As we will argue later, the only reserve price consistent with equilibrium behavior for prices below $\underline{\theta}$ is $\underline{\theta}$. Thus the informed investor's equilibrium demand will be strictly positive for $p \in (0, \underline{\theta}]$.

In contrast to the financial investors, noise traders' demand is by construction inelastic to the seller's second-stage decision. Their demand is simply a hyperbolic function in p for any $x > 0$ and contains no information about the state of the world.

To determine the optimal reserve price, the seller is interested in the realization of $\widetilde{\theta}$ but not in the amount of noise $\widetilde{x}$. Both random variables affect the price though. She knows which parties exhibit a positive demand in equilibrium, given a certain price. To be able to update her beliefs about the probability of a high θ, we need that at least one party's demand is elastic with respect to $\widetilde{\theta}$. If the seller for example observes a price below the cut-off point, no information is revealed since the noise traders' demand does not depend on the state of the world and informed traders do not buy in any case. Prices outside the interval $[\underline{\theta}, \overline{\theta}]$ reveal no information either. If p is smaller than the lower bound, informed agents demand in both states of the world while for prices exceeding the upper bound, their demand is always zero. For prices in $(p^*, \overline{\theta})$ the seller has to deliberate about which probability mass to put on combinations of (x, θ) which are consistent with the observed outcome. To be able to run through this procedure we first have to determine the equilibrium relationship of the price with the unknown variables $P(x, \theta)$.

In the next section we derive the equilibrium in the financial market given that the seller uses the cut off rule in the second stage.

2.3 First stage

In the first stage informed investors submit demand schedules $x_i(\theta, p)$ given the expected value of $\widetilde{v}$ from the second stage. Recall that these demand schedules can be any piecewise continuous correspondence mapping prices p into non empty subsets of the interval $[0, +\infty)$. An auctioneer receives the demand schedules and calculates the set of market clearing prices and corresponding allocations as described in Section 2. This procedure gives a well defined price for any pair (x, θ), which will be denoted as $P(x, \theta)$.[21]

Notice that in our setup a positive and finite equilibrium price always exists due to the following. Noise trade is always positive but monotonically continuously falling in the price, asymptotically reaching zero as price goes to infinity[22] and going to infinity as p goes to zero. The informed buyer's demand is an upper hemi continuous correspondence in $(0, +\infty)$

[21] Note that due to the liquidity constraints there is no need to account for the case of infinite demands as in the Kyle model.

[22] Note that the liquidity constraints actually imply demand becomes zero for some large enough price.

except for the upward jump in p^*, going to zero as price goes to infinity. Also recall that short selling is not allowed. Given these facts it is easy to verify that aggregate demand is surjective in $(0, +\infty)$, thus if supply is constant and non trivial or infinite, there is always a positive, finite price at which demand equals supply.

In equilibrium all informed agents maximize utility given the demand functions of the others and the information revealed by the resulting price. Formally, we have

Definition 1 *A symmetric Nash equilibrium in trading strategies is defined as a function* $x_i(\theta, p)$ *such that* x_i *solves the maximization problem of the agents conditional upon their information:*

$$\max_{x_i} E_{x,\theta}[\widetilde{v}]x_i + (1 - px_i)$$

We have assumed there is a continuum of financial investors who are price takers. In contrast to the one-stage model the asset's return to the financial investors is determined endogenously. It depends on the realization of the random variable $\widetilde{\theta}$ but also on the resulting price in the aftermarket, $\widetilde{v}$. Recall that in case the asset is not sold, it has to be liquidated[23].

In equilibrium, the informed financial investors correctly anticipate the seller's reserve price decision when the market clears at price p. We have assumed that these agents are perfectly informed about the value to the investor, therefore they can foresee whether a transaction will take place in the aftermarket. If the seller asks for a high price $\overline{\theta}$, no transaction will take place when the true value is low. This leads to a liquidation and zero payoff. On the other hand, a reserve price of $\underline{\theta}$ ensures an efficient transaction but does not extract the full surplus if the control premium is high. The reduced form value function to the informed financial investors is:

$$v(p, \theta) = \begin{cases} \overline{\theta} & \text{if} \quad r^*(p) = \overline{\theta} \text{ and } \theta = \overline{\theta} \\ \underline{\theta} & \text{if} \quad r^*(p) = \underline{\theta} \\ 0 & \text{else} \end{cases} \tag{3}$$

We are now equipped with all the ingredients to solve stage 1. Recall that the financial investors are risk neutral and liquidity constrained. Optimal behavior –as defined above– requires that they invest all available funds in the asset with the highest return. Due to the

[23] At this point one has to mention the Coase Conjecture. To ensure the credibility of the reserve price, the seller could delegate the sale in stage 2 to an agent who is committed to the strategy of selling for the ex-ante optimal reserve price or liquidating the asset otherwise.

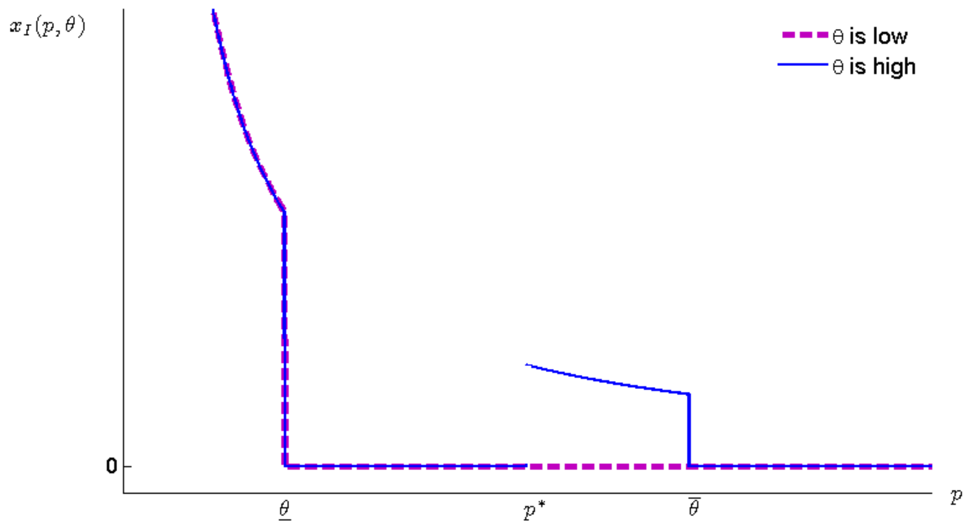


Figure 1: Demand of the informed investors depending on price, when the value of the company is high/low.

riskless bond having zero yield, demand for stock will be positive as long its value exceeds its price. Thus, the aggregate demand of the informed sector, denoted by x_I, becomes:

$$x_I(p,\theta) = \begin{cases} \frac{1}{p} & \text{if} \quad p < v(p,\theta) \\ \left[0, \frac{1}{p}\right] & \text{if} \quad p = v(p,\theta) \\ 0 & \text{if} \quad p > v(p,\theta) \end{cases} \tag{4}$$

Note that no individual informed agent has an incentive to deviate from this strategy as she can not influence the equilibrium price.

Let $P(x,\theta;p^*)$ denote the market-clearing price for a pair (x,θ) given a cut-off point p^*. We can now calculate the equilibrium price function:

Proposition 1 *Given that the seller issues a fraction λ of the asset and determines her optimal offer to the buyer by a cut-off rule $p^* \in (\underline{\theta},\overline{\theta})$. Then there exists an equilibrium in the asset market, where*

1. *Aggregate demand is given by (A.2) and (4).*

2. The market-clearing price is

$$P(x,\underline{\theta};p^*) = \begin{cases} \frac{wx}{\lambda} & \text{if} \quad x > \frac{\lambda\underline{\theta}}{w} \\ \underline{\theta} & \text{if} \quad x \in \left[\frac{\lambda\underline{\theta}-1}{w}, \frac{\lambda\underline{\theta}}{w}\right] \\ \frac{wx+1}{\lambda} & \text{if} \quad x < \frac{\lambda\underline{\theta}-1}{w} \end{cases} \tag{5}$$

and

$$P(x,\overline{\theta};p^*) = \begin{cases} \frac{wx}{\lambda} & \text{if} \quad x > \frac{\lambda\overline{\theta}}{w} \\ \overline{\theta} & \text{if} \quad x \in \left[\frac{\lambda\overline{\theta}-1}{w}, \frac{\lambda\overline{\theta}}{w}\right] \\ \frac{wx+1}{\lambda} & \text{if} \quad x \in \left(\frac{\lambda p^*-1}{w}, \frac{\lambda\overline{\theta}-1}{w}\right) \\ \frac{wx}{\lambda} & \text{if} \quad x \in \left(\min\{\frac{\lambda\underline{\theta}}{w}, \frac{\lambda p^*-1}{w}\}, \frac{\lambda p^*-1}{w}\right] \\ \underline{\theta} & \text{if} \quad x \in \left[\frac{\lambda\underline{\theta}-1}{w}, \min\{\frac{\lambda\underline{\theta}}{w}, \frac{\lambda p^*-1}{w}\}\right] \\ \frac{wx+1}{\lambda} & \text{if} \quad x < \frac{\lambda\underline{\theta}-1}{w} \end{cases} \tag{6}$$

This price function, illustrated in Figure 2, is constructed by solving the market clearing condition, given the strategies of the informed investors and the demand of the noise traders, separately for the cases that θ is high or low[24]. For example, in case x is greater than $\frac{\lambda\overline{\theta}}{w}$, then, demand from the noise traders alone pushes the price above $\overline{\theta}$ in which case the strategic investors do not buy any shares. Thus the price is equal to the trader wealth wx divided by the supplied shares λ. We proceed similarly for the other possible values of the price.

Note that if the cut-off price p^* is sufficiently close to $\underline{\theta}$ and the realized premium is high then there is an interval of x, each consistent with two different prices: in the low-price case, only noise traders demand while at the higher price also informed traders participate.[25] We have constructed the function in proposition 1 by selecting the higher of the two prices.

After the seller observes a price p she can update her beliefs about the state of the world. Using (5) and (6), she can calculate the posterior probability $\Pr(\overline{\theta}|p,p^*)$ of the true state θ being high in the following way:

If the seller observes a market-clearing price $p\prime$, then inverting the family of price functions $P(\theta)$ gives her two realizations of $\widetilde{x}$ which are consistent with equilibrium, $x_1(p\prime)$ and $x_2(p\prime)$. From the ex ante distribution of the noise she can infer how likely it is that this price was generated by a high θ or high noise trade. On the contrary, $p\prime\prime$ contains no such information

[24] We limit our analysis to $\underline{\theta} > 1/\lambda$ in order to avoid additional case distinctions.

[25] This is a consequence of the non-monotonic demand function of the informed financial investors.

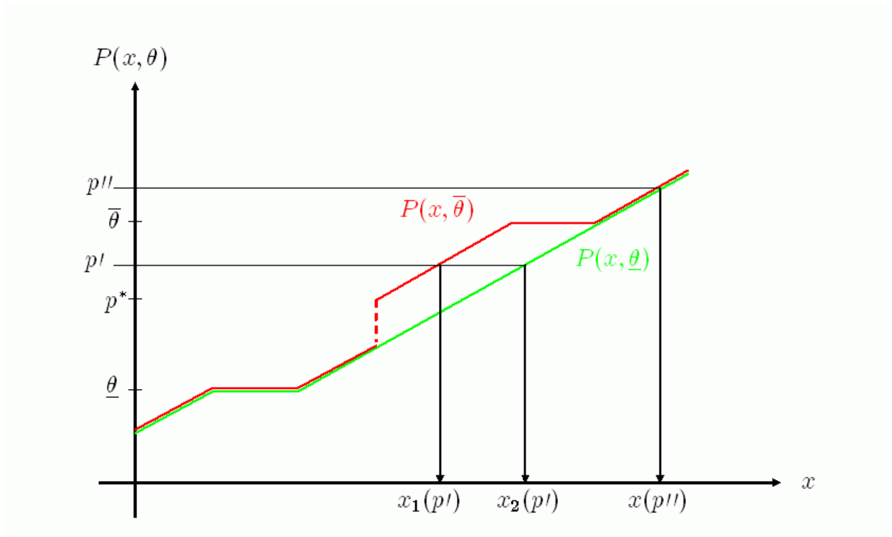


Figure 2: Determination of posterior beliefs.

since it is associated with a single realization of $\widetilde{x}$. Full information revelation is only possible for prices which correspond to just one state of the world. Such a price can be found in Figure 2 as a point on the y axis from which a line parallel to the x axis intersects with only one of the two depicted curves.

Algebraically, the application of Bayes' rule[26] leads to the following posterior probability for a high control premium:

$$\Pr(\overline{\theta}|p,p^*) = \begin{cases} \sigma & \text{if} \quad p > \overline{\theta} \\ 1 & \text{if} \quad p = \overline{\theta} \\ \xi(p) & \text{if} \quad p \in (p^*, \overline{\theta}) \\ 0 & \text{if} \quad p \in (p^* - \frac{1}{\lambda}, p^*] \\ \sigma & \text{if} \quad p \leq p^* - \frac{1}{\lambda} \end{cases} \tag{7}$$

26

$$\begin{aligned} \Pr(\overline{\theta}|p) &= \frac{\Pr(\overline{\theta} \cap p)}{\Pr(p)} = \frac{\Pr(\overline{\theta})\Pr(p|\overline{\theta})}{\Pr(p)} \\ &= \frac{\sigma \Pr(p|\overline{\theta})}{\sigma \Pr(p|\overline{\theta}) + (1-\sigma)\Pr(p|\underline{\theta})} = \frac{\sigma}{\sigma + (1-\sigma)\frac{f(\lambda p/w)}{f((\lambda p - 1)/w)}} \end{aligned}$$

with $\xi(p) \equiv \sigma\left[\sigma + (1-\sigma)\frac{f(\lambda p/w)}{f((\lambda p-1)/w)}\right]^{-1}$.

Prices outside $(p^* - \frac{1}{\lambda}, \overline{\theta}]$ contain no information. Their occurrence stems from high and low realizations of the noise trade component $\widetilde{x}$ respectively. In contrast, if the price hits the upper bound, information is fully revealed due to the indifference of the informed traders. At this price they will demand any amount of shares, which leads to a range of realizations of $\widetilde{x}$ that support $P = \overline{\theta}$. However, when the state of the world is $\underline{\theta}$ the value of the asset is strictly lower than its price. Thus demand is solely driven by noise traders which leads to exactly one x where demand equals supply[27]. Only for prices in $(p^* - \frac{1}{\lambda}, \overline{\theta}]$ can the seller actually update her beliefs.

Now we can proceed to characterize the equilibrium of the game.

2.4 Existence of equilibrium

In the next proposition we show which conditions ensure the existence of an equilibrium.

Proposition 2 *Suppose the seller floats a fraction λ of the asset in the financial market. If the distribution f is log-concave there exists a cut-off equilibrium $p^* \in [\underline{\theta}, \overline{\theta}]$ such that the optimal tioli-offer to the buyer is*

$$r^* = \begin{cases} \overline{\theta} & \textit{if} \quad p \in [p^*, \overline{\theta}] \\ \underline{\theta} & \textit{else} \end{cases}$$

Proof. See appendix. ■

Lemma 3 gives condidtions for which we have an interior solution in this equilibrium. Common cases that fulfill log-concavity include the Normal, Poisson, and triangle distributions (see [4] for a detailed survey). Note that for the existence of a cut-off equilibrium log-concavity is sufficient but not necessary. Necessary is that once the posterior probability exceeds the indifference point $\underline{\theta}/\overline{\theta}$ it remains above it. When this monotonous ratio condition is violated, any number of cutoff points is possible. Also note that the equilibrium discussed here is not unique. In fact in Section 3.2 we discuss an additional equilibrium and ways to discard it.

[27] This results hinges upon the distribution of $\widetilde{x}$. Since we modeled it as a continuous random variable, every realization of $\widetilde{x}$ which could cause the price to correspond to $\overline{\theta}$ when this is not the true state of the world is a zero-probability event.

In the next section we compare the expected revenue generated by the two-stage mechanism with two obvious alternatives: a sealed-bid auction with optimal reserve price and a public offering of 100% of the shares.

2.5 Revenue comparison

Let us start with the calculation of the expected revenue in an auction for the whole asset. Recall that the financial investors are financially constrained. Given that their endowment is lower than the lowest possible private value of the strategic investor $\underline{\theta}$, they can never influence the outcome of an auction with a reserve price that is set optimally to extract rents from the strategic investor.[28] Thus the task for the seller reduces to an optimal indivisible good auction with just one strategic investor, which amounts simply to an optimal take-it-or-leave-it offer.

The prior distribution of the company value to the strategic investor is, according to (A.1), such that the seller optimally offers $\underline{\theta}$ and the buyer always accepts. Therefore, the expected revenue from the one-stage auction yields $\underline{\theta}$ in equilibrium.

$$\Pi_{OA} = \underline{\theta} \tag{8}$$

In the case of an IPO for 100% of the shares, as we have explained in the previous section, the strategic investor will not participate as she can purchase a majority stake later by making a minimal offer to the small investors. In contrast to the noise traders, informed traders anticipate this behavior correctly and demand no stocks at any positive price. The only demand component which drives the price above zero is the noise trade. Therefore, the expected revenue for the seller equals the total expected wealth of the noise traders

$$\Pi_{IPO} = w \int_0^1 x f(x) dx \tag{9}$$

The advantage of going public over an optimal tioli-offer is obvious: the seller can fleece noise traders. If sufficiently high probability mass is on realizations below $\underline{\theta}/w$ then an optimal auction outperforms the wholesale IPO.[29]

[28] The optimal reserve price will always be $r^* \geqslant \underline{\theta}$. Recall that the endowment of the individual informed investor is assumed to be lower than $\underline{\theta}$.

[29] If an auction with optimal reserve price is chosen by the seller, this results in a market with a monopolist

If the seller chooses the two-stage mechanism instead and issues a fraction λ of the asset, her expected revenue consists of the expected IPO price (Π_{PIPO}) and the revenue from the subsequent bargaining (Π_{RP}):

$$\Pi_{TS} = \lambda \Pi_{PIPO} + (1-\lambda)\,\Pi_{RP} \tag{10}$$

The IPO price serves as a signal for the seller to extract information from the informed financial investors and thus to update her beliefs. She will only switch from $\underline{\theta}$ to $\overline{\theta}$ if a higher reserve price generates a higher expected revenue. It immediately follows that in the two-stage mechanism she will be better off than in the optimal auction regarding the non-issued fraction $(1-\lambda)$. If an interior cut-off point exists then the expected revenue from the second stage can be written as

$$\Pi_{RP} = \int_{[0,p^*]\cup(\overline{\theta},\infty)} \underline{\theta} g(p)dp + \int_{(p^*,\overline{\theta}]} \overline{\theta}\,\Pr(\overline{\theta}|p)g(p)dp$$

with $g(p)$ denoting the distribution of prices in equilibrium.

Let us compare this to the expected revenue of the wholesale auction, which is the lower realization of the company value:

$$\Pi_{RP} > \Pi_{OA}$$

$$\Leftrightarrow \int_{[0,p^*]\cup(\overline{\theta},\infty)} \underline{\theta} g(p)dp + \int_{(p^*,\overline{\theta}]} \overline{\theta}\,\Pr(\overline{\theta}|p)g(p)dp > \underline{\theta}$$

Since $[0,\infty)$ covers the whole support of prices in equilibrium, the first integral can be rewritten as one minus $\underline{\theta}$ times the probability of prices in $[p^*,\overline{\theta}]$. Therefore we get

$$\int_{(p^*,\overline{\theta}]} \overline{\theta}\,\Pr(\overline{\theta}|p)g(p)dp > \int_{(p^*,\overline{\theta}]} \underline{\theta} g(p)dp$$

Sufficient for this inequality to hold is that the integrand in the left part is point-wise bigger than the integrand to the right, i.e. for all $p \in (p^*,\overline{\theta}]$

facing a monopsonist. Such a case should leave the buyer worse off compared to bargaining with a continuum of agents who all possess the same outside option: liquidation of the asset. We claim that any other reasonable bargaining specification should not alter our results qualitatively but would make the analysis more cumbersome. See section 4 for further discussion of this issue.

$$\overline{\theta}\Pr(\overline{\theta}|p)g(p) > \underline{\theta}g(p) \Leftrightarrow \Pr(\overline{\theta}|p) > \underline{\theta}/\overline{\theta}$$

which follows from Lemma 3 (see appendix). Thus we have shown the revenue per share in the second stage is always higher than in the auction.

Now, it is easy to show that the revenue in the first stage $\lambda\Pi_{PIPO}$ is always greater than the revenue in the full IPO Π_{IPO}. Observe that the revenue in an IPO without a second stage, where only the noise traders participate will always be the same, independently of λ.This is due to the fact that the noise traders always spend their whole wealth so that the price elasticity of demand is always -1, a higher supply λ leads to a one to one reduction in the price and vice versa. This implies $\lambda P_{IPO}(\lambda) = \Pi_{IPO}$.Given that the only difference between the first stage of the two stage mechanism and an IPO without a second stage is the possible extra demand coming from the informed traders, the revenue $\lambda\Pi_{PIPO}$ will always be higher than λP_{IPO} which in turn is equal to the revenue in the full IPO. Thus we see the revenue in the first stage is always higher than the revenue in a full IPO. Combining these two observations we have following result:

Conclusion 1 *The two stage mechanism performs always better than the full IPO and the optimal auction. The ranking between the optimal auction and the full IPO is ambiguous and depends on the size of the noise trade $E[x]w$ and the prior σ.The higher the noise trade, the more attractive the IPO becomes while the opposite is true when the prior σ becomes higher.*

The first part follows from the discussion above and the fact that λ is chosen optimally.[30] The revenue in the two stage mechanism is a convex combination of two elements that are always respectively higher than the revenues in the two other mechanisms. Thus an optimal λ leads to always higher revenues than each of the two other mechanisms. Actually, as we have shown, the two stage mechanism is better than the the full IPO for any possible $\lambda < 0.5$. In both mechanisms the seller extracts all the noise traders' wealth, but in the two stage one she can also extract a part of the strategic investor's revenue.

A natural question arises as to why a full IPO is not a good alternative, especially if we think that a two stage mechanism is more complicated and probably more costly in reality. The answer lies in the incentives of the financial investors. In the absence of a second stage

[30] We have shown that there exists an optimal λ in $[0, 0.5)$. However, we do not derive the optimal λ explicitely, since it is prohibitively complex to provide an analytical solution for general distribution functions. For an explicit determination for a given family of distributions be referred to the numerical example.

the financial investors cannot expect a resale and with it a sell-out rule to apply[31]. They cannot try to buy a majority stake themselves in order to resell it, as they are liquidity constrained. Thus their valuation of the shares equal just the cash flow part. The strategic buyer has no incentive to participate in a full IPO either, as he can always wait and make an offer to the small investors after the IPO. Recall there is a continuum of them so they have no bargaining power and the strategic investors will just pay their reserve price, which equals the common part of the valuation of the company.

Another remark is in order here. In real markets IPOs bring along significant underwriting and marketing[32] costs. Assuming a fixed cost of underwriting, the auction can actually be more interesting to the seller than the two stage mechanism. Additionally, having several potential acquirers improves the performance of the auction. To illustrate our results we calculate a simple numerical example and present some comparative statics in the next section.

3 Numerical Example

In this section we will use a specific density to illustrate our results and conduct some comparative statics. As we have noted, there is a wide range of distributions that fulfill the log-concavity assumption. We choose a simple bounded distribution with enough versatility. *Kumaraswamy's* double bounded distribution [14] has a simple closed form for both its PDF and CDF. In the simplest case, we can take the bounds to be $x \in [0, 1]$ in which case the probability density function is:

$$f(x) = abx^{a-1}\left(1 - x^a\right)^{b-1}$$

The mode for $a, b > 1$ is given by

$$\left(\tfrac{a-1}{ab-1}\right)^{1/a}$$

Next figure plots the distribution for $a = 1.5$ and $b = 1.1$, 2, 4, 8, 15 and 20.

[31] In all IPOs no investor gets more than a limited percentage of the company. Buying a majority stake is usually impossible. Examining the optimality of these rules is outside the scope of this paper. However it should be noted that our framework can be useful in analyzing such regulations.

[32] Actually part of the job of the underwriting banks is to raise the amount of noise trade!

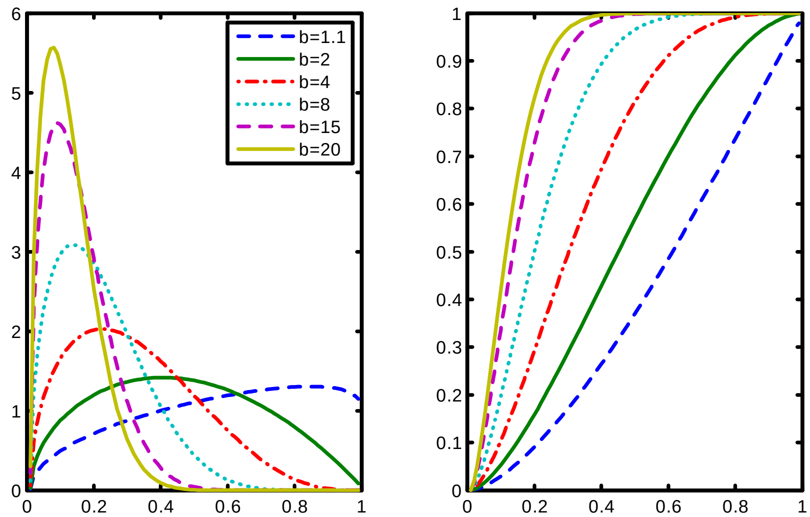
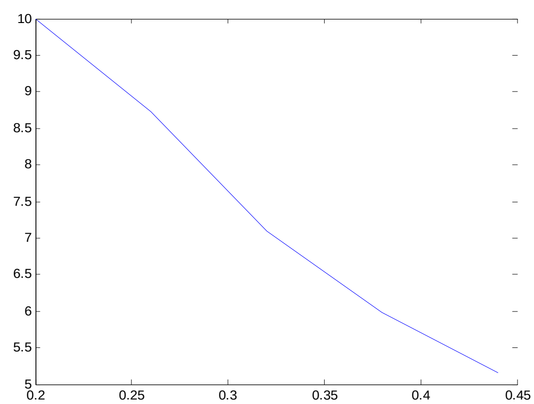
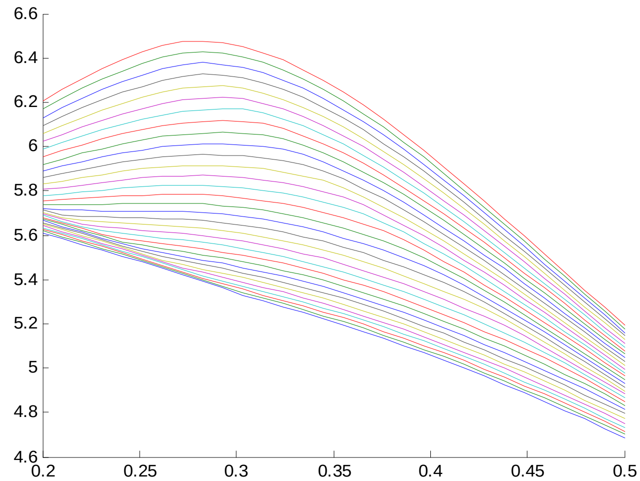


Figure 3: Pdf and cdf of the noise trade.

Fix $a = 1.5$ and $b = 8$. For a given σ we can now find the optimal cutoff p^* depending on the emission size λ. The optimal cutoff is going to fall in λ, as the increased percentage of shares offered makes the noise trader component of the demand less important and thus the price more informative.



Given this we can calculate the optimal λ by maximizing expected revenues. In following plot we draw the expected revenue for a series of $\sigma \in [0.2, 0.5]$. Observe for high σ the revenue has an inverse U shape so λ^* is an interior solution, while for low values we get a corner solution; the seller tries to issue an infinitesimally small share in the first stage IPO.



The revenue comparison, as we described in the previous section, is strict. The revenue of the two stage mechanism depends on σ but is always higher than the revenue in two other mechanisms which are independent of σ. The auction is better than the full IPO only when there is not much noise (which means when $E[x]$ is low, or when w is low).

3.1 Comparative statics

In this section we evaluate the model for different values of the parameter b for the noise trade distribution. Figure 4 plots the optimal IPO issue size λ^* against the prior σ for the different distributions. We assume there is an exogenously given minimum $\lambda_{\min} = 0.2$. Observe that for $b = 4$ and distributions that are more left skewed (that is have a lower b) we have a corner solution, λ is always low. In general there is no clear relationship between the shape of the distribution and the optimal λ, independently of σ. For low values of σ right skewed distributions lead to a higher λ, while for high values of σ the opposite is true.

In Figure 5 we plot the optimal cutoffs p^* depending on σ for the 6 different parameterizations. Here a clear relationship can be seen. Again for $b = 4$ we have a corner solution, but for higher b (more right skewed distributions) the cutoff is falling.

It is also worth investigating what happens for different sizes of the noise trade, that is when we vary the noise trader wealth w. We find that the effect of a change in w affects the optimal cutoff and optimal λ in a very similar way as a change in the distribution parameters. That is, a higher w makes the cutoff p^* always smaller, but the effect on λ is ambiguous and it depends on σ.

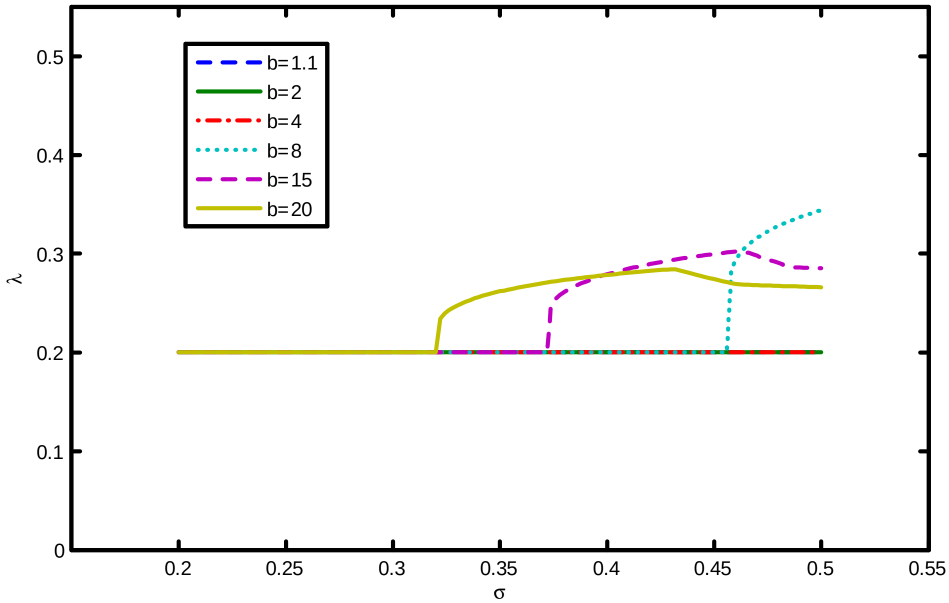


Figure 4: Optimal λ depending on σ.

4 Discussion and Extensions

In this section we will discuss some characteristics of the model and present possible extensions.

An unusual result of our model is that the informed investors benefit from more informative market prices and thus have a preference for low amounts of noise trade. This is in contrast to models like Kyle (1989) where the noise traders are exploited by the informed, who as a consequence prefer markets with plenty of noise trade. The intuition is that in our model informed traders want to signal the value of the premium to the seller accurately, in order for him to set a more beneficial reserve price. Noise traders are only hindering this task.

Also worth noting is the seemingly paradoxical result that in our model more (correct) information can lead to less efficiency. The most efficient reserve price is one set at the lowest value of the premium, where the company is sold for sure. If the seller chooses direct bargaining she sets such a reserve price, given our assumption on the prior. However when the seller uses the two stage mechanism she gets more information and updates her prior.

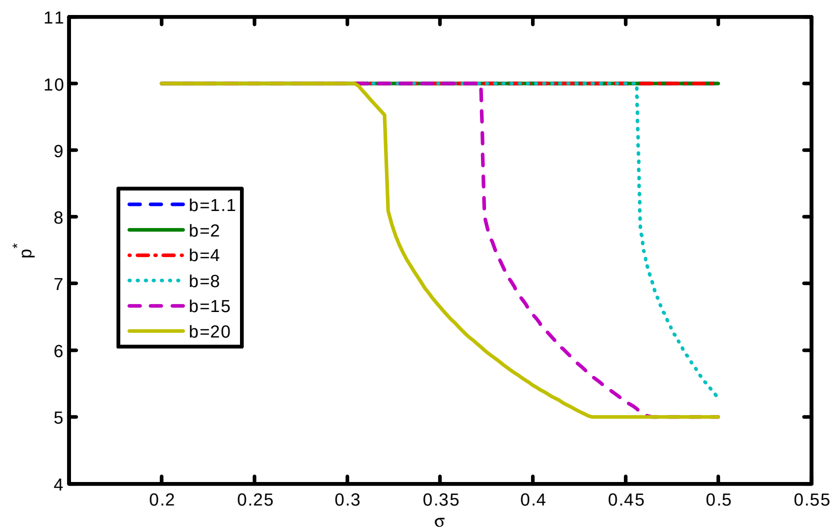


Figure 5: Cutoff p^* depending on σ.

The updated probability of a high premium can induce her to raise the reserve price even if the premium is not actually high! There is an intuitive parallel we can draw with models of imperfect competition where market power in general lowers efficiency. In our model information gives the seller an advantage in the bargaining game which raises his expected revenue but is possibly detrimental for overall efficiency.

Another interesting property of the equilibrium is that it involves a demand function which is discontinuous and non-monotonic in the price. This is due to the cutoff strategy of the seller. Even when the value of the premium is high, informed traders are interested in buying only if the price is above some limit (in our model p^*). For prices under this limit they expect the seller to set a low reserve price in the second stage and thus do not want to pay more than $\underline{\theta}$ for the shares. This characteristic of the demand function is a robust feature of the two stage mechanism and will not fade away if we have more states of the world.

Lastly, a natural extension would be a deeper modelling of the information gathering process. Endogenous information acquisition, where the investors can buy degrees of precision would add generality to our model. Naturally, this addition will make the model very complicated, as we can see in the following section on strategic uninformed traders, but is a promising avenue for future research.

4.1 Strategic uninformed traders

So far we have assumed there is a continuum of informed traders and an exogenous amount of noise trade, thus there are no real strategic players in the first stage market. In the other extreme case where all agents act strategically there would actually be no uncertainty, once a market-clearing price is observed, as we have seen in Section 2. The non-strategic noise trade component in aggregated demand is responsible for shading the true θ.

Now, what happens if there is still some noise trade but only a fraction z informed about the control premium? Such a structure is consistent with the majority of financial market models.

Starting from the setup we analyzed before we add uninformed investors with total wealth w_U. Recall that total initial endowment of the informed agents was normalized to one. Since all agents are identical except for the information they hold we can express the fraction z of the informed investors in the following way:

$$z = \frac{1}{1 + w_U}$$

Uninformed investors face a similar problem than the seller when they observe a market-clearing price: to which extent is the price driven by noise traders and by informed agents respectively? To determine how their presence alters the outcome we proceed in the following way: we take the original equilibrium and analyze optimal behavior of the uninformed when they enter this market. Demand schedules are submitted simultaneously. Thus in the second step we have to check whether this behavior is consistent with an equilibrium and how it affects the other market participants' strategies. A priori we do not know if the seller's cut-off point is a function of z and how it affects informed investors' optimal demand.

Recall that the inner optimal cut-off point was implicitly defined as the price p^* such that

$$\Pr\left(\overline{\theta}|P = p^*; p^*\right) = \frac{\theta}{\overline{\overline{\theta}}}$$

given the conditions stated in Lemma 3 (see appendix) are met.

Informed agents invest all their wealth in the asset with the highest yield, i.e. they buy stocks whenever the price is lower than their value. Uninformed traders conduct a similar calculation. Since they cannot observe θ, they form conditional expectation of $\widetilde{v}$ (which is a function of θ) based on the price.

Let x_u denote the optimal demand schedule, represented by

$$x_U(p) = \begin{cases} \frac{w_U}{p} & \text{if} \quad p < E[\widetilde{v}|P = p] \\ \left[0, \frac{w_U}{p}\right] & \text{if} \quad p = E[\widetilde{v}|P = p] \\ 0 & \text{if} \quad p > E[\widetilde{v}|P = p] \end{cases} \tag{11}$$

Let us ignore the impact of this additional demand component on the market-clearing price for a moment. It is therefore very easy to determine the net benefits as a function of the price. If the price is smaller than $\underline{\theta}$ we know that no info is revealed but net benefits are positive. Unless the price exceeds the cut-off point p^*, the seller's action results in a second-stage price of $\underline{\theta}$ and demand will thus be zero in $(\underline{\theta},\ p^*)$. As the price approaches the upper realization net benefits converge to

$$\Pr(\overline{\theta}|p;p^*)\overline{\theta} - \overline{\theta} \leq 0$$

There are two possible cases how net benefits evolve between the cut-off point and the high realization as demonstrated by the next figure.

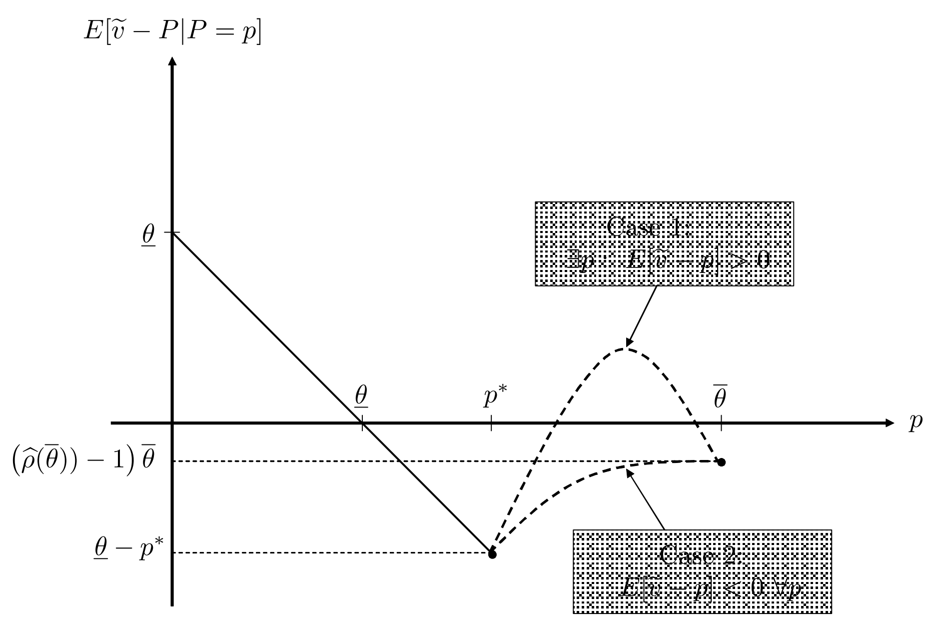


Figure 4: Net value for uninformed investors.

We know from the previous analysis that the conditional probability of a high premium is a continuous function in the price in the upper interval. Therefore, net benefits are also a continuous function and we can distinguish two generic cases:

1. There exists a non-degenerate price interval in the interior of $(p^*, \overline{\theta})$ such that net benefits are strictly positive.

2. Net benefits are negative for all prices greater than $\underline{\theta}$.

In the latter case the only change occurs to the price function for the interval below $\underline{\theta}$. Lemma 3 remains valid. On the other hand in the first case a general analytical result is not attainable.

If a cut-off exists then prices where the uninformed exhibit positive demand are always a strict subset of prices for which the informed investors buy. This means that being informed is truly better independent of the fraction z. If the cost of information acquisition is small enough all investors will be informed in equilibrium. This resembles the situation we analyzed throughout the paper.

4.2 More than two states of the world and zero information equilibria

The equilibrium in Proposition 1 is informative, in the sense that the speculators reveal their private information and the noise traders are the only hindrance to full information revelation. In case we have more states of the world such an equilibrium continues to exist. Assume for example there are three states of the world θ_l, θ_m and θ_h with a vector of prior probabilities σ. Let ξ^* be a vector of posterior probabilities for each state of the world (ξ_l, ξ_m, ξ_h) such that, for all $\xi^{'}$ where θ_m and/or θ_h are more likely than in ξ^*, the seller chooses a reserve price of θ_m and conversely for all $\xi^{''}$ that assign a lower probability weight on θ_m or θ_h than in ξ^*, θ_l is chosen. Define ξ^{**} similarly as the vector of probabilities so that the seller switches from θ_m to θ_h. A sufficient condition for the informative equilibrium is that the posterior ξ "crosses" ξ^* and ξ^{**} once and only once, meaning that there is a price such that $\xi_m + \xi_h < \xi_m^* + \xi_h^*$ for all lower prices and for all higher prices $\xi_m + \xi_h > \xi_m^* + \xi_h^*$. The analogous must be true for ξ_h and ξ_h^*. There are parameters for which these conditions hold.

However there also exist zero information equilibria. Suppose for example the informed speculators believe that the company will be sold for θ_l and no other speculator will buy shares. Indendently of the number of possible states of the world, if the prior is such that without additional information the seller chooses a reserve price equal to θ_l, such beliefs are self fulfilling. The speculators buy no shares and the seller does not update the prior. This actually leads to a reserve price of θ_l and in this equilibrium the market price reveals no information whatsoever.

Note there are also partially informative equilibria of this type. Assume the speculators believe the company will never be sold for more than θ_m. Then, if the above conditions hold, the speculators will sometimes be able -depending on the noise- to signal that θ is not low by buying shares at the appropriate prices. However they will never buy above θ_m and the seller will never ask for a reserve price higher than this[33]. Thus this equilibrium is also self fulfilling and not all private information is included in the market price.

A way to discard these equilibria is market power. If one informed speculator has enough market power to move the market price and reveal his information, subgame perfection requires that the seller responds by choosing the appropriate reserve price. Since the speculator always wants the reserve price to reflect his information it is optimal for him to indeed buy shares and signal his information. Thus with market power zero information equilibria do not exist.

4.3 Bargaining Power

A point that should be discussed is the extreme bargaining power we attribute to the seller, by allowing him to make a take it or leave it offer. Obviously, if we move to the other extreme and the expected allocation does not depend on the seller's information, e.g. if the seller has no bargaining power, the seller will not want to acquire information and the two stage mechanism is rendered useless. Arguably such a case will be very rare.

A more plausible configuration is a bargaining model where the seller has no full bargaining power but the expected allocation still depends on the information the agents possess. For example in Rubinstein and Wollinsky (1985) one of the two parties is selected randomly to propose a split of the gains from trade. In our framework this means the seller will want to acquire information, to choose a better proposal in case she is selected to make an offer. In the simplest case where the bargaining game is played just once and rejection of the offer

[33] There is an additional condition on the prior probabilities and on the $\theta's$ for this equilibrium to exist. It must be that the seller wants to set a medium reserve price whenever he cannot distinguish between demand coming from a high state of the world and demand coming from a medium state.

leads to liquidation the basic results of our model hold. There is no qualitative change, just a quantitative shift in the parameter constellations where a two-stage mechanism dominates the others.

4.4 Information structure

As a last note, we would like to point out that the special information structure of our model can have an alternative interpretation. Assume there is no control premium, but the value of a company stems just from its discounted stream of dividends. Investors, however, differ in their prognosis of events that can have an industry-wide effect on all firms in the relevant market. Insiders, such as companies in the field or industry-specific analysts, can be assumed to have a superior prediction of the future. On the other hand, governments or large non-focused corporations do not have access to such information. When these large agents try to sell a firm, they can benefit from the information of the small players in a very similar way to the one described above[34].

5 Conclusions

Our analysis shows that small financial investors can help a seller extract higher rents from the potential buyers in an IPO. The use of a two-stage mechanism for this purpose yields a higher revenue, under some conditions, than a simple IPO or an optimal auction. Important parameters for a seller contemplating a decision between the three alternatives, are the amount of noise trading in the market, the number of financial investors that can be informed of the value of the company and the number of strategic investors who are possibly interested in acquiring it. With a large number of strategic investors the advantage that a reserve price can give, becomes quite small. On the other hand a large number of informed investors makes the information aggregation through the IPO stronger and the two-stage mechanism more attractive. A great amount of noise trade can have ambivalent effects. It will make the information aggregation in the IPO worse. On the other hand, it will raise the demand for the shares and thus raise the seller's revenues. When there is a lot of noise

[34]In such a specification, the value of the company to potential owners would be an affiliated/common value. Multiple strategic buyers with affiliated values would greatly complicate the analysis without adding to the intuition.

trade, as might be the case in a bull market, the best option for the seller is to sell the entire company through a full IPO.

In general we think our framework is a useful tool to discuss the ongoing privatization programs all around the World and the sale of divisions by big corporations. This paper demonstrates that the presence of informed but liquidity constrained financial investors, can explain in part the broad usage of IPOs in the field. Additionally we show that sell-out rules are important for the presence of small informed investors and thus for the informational content of prices in a stockmarket. Our analysis should thus interest regulators contemplating plans to impose sell-out rules in financial markets, such as the recently voted EU takeover directive.

Finally, our results can be useful to analyze the informational content in the price of a company's shares in the stock market once a takeover attempt has been announced. Under some conditions the share price will be an accurate signal for the valuation of the target company. On the other hand, an unfavorable result is also possible. The market can be stuck in a zero information equilibrium and the market prices only reflect noise. However, when some informed agents have market power these zero information equilibria cease to exist.

6 Appendix: Proofs

We start with the proof of Proposition 2:

Proof. Recall from (1) that the seller sets a reserve price $r = \overline{\theta}$ whenever the posterior probability of a high value exceeds $\underline{\theta}/\overline{\theta}$ and $r = \underline{\theta}$ otherwise. According to (7), the posterior probability can only exceed the ratio of the two realizations for prices in $[\underline{\theta}, \overline{\theta}]$. Thus, an optimal cut-off has to lie in this interval. Choosing any cut-off point, the resulting posterior will always be, by construction, consistent with optimal behavior for prices below the cut-off: the resulting posterior is either σ or 0, and by assumption A.1 lower than $\underline{\theta}/\overline{\theta}$. This cut-off is suboptimal if there are prices above p^* for which the posterior $\xi(p)$ induces a low instead of a high offer, i.e. $\xi(p) < \underline{\theta}/\overline{\theta}$.

Suppose $\xi(p)$ is monotonically increasing. Then there are three possible cases:

1. $\xi(p) > \underline{\theta}/\overline{\theta}\ \forall p \in [\underline{\theta}, \overline{\theta}]$: the posterior always exceeds the ratio and the optimal cut-off is $\underline{\theta}$ (high offer at all prices)

2. $\xi(p) < \underline{\theta}/\overline{\theta}\ \forall p \in [\underline{\theta}, \overline{\theta}]$: the posterior lies always below the ratio and the optimal cut-off is $\overline{\theta}$ (low offer at all prices)

3. $\exists! p^* \in [\underline{\theta}, \overline{\theta}] : \xi(p^*) = \underline{\theta}/\overline{\theta}$: the posterior and the horizontal line at $\underline{\theta}/\overline{\theta}$ intersect just once. Uniqueness and existence are guaranteed by strict monotonicity and continuity of ξ in p. The latter property follows from the differentiability of f.

What remains to be shown is that the log-concavity of f is sufficient for the monotonicity of ξ.

$$\frac{\partial \xi(p)}{\partial p} = -\sigma(1-\sigma)\frac{\lambda}{w}\frac{f'(\lambda p/w) f((\lambda p - 1)/w) - f'((\lambda p - 1)/w) f(\lambda p/w) f((\lambda p - 1)/w)^{-2}}{\left(\sigma + (1-\sigma)\frac{f(\lambda p/w)}{f((\lambda p-1)/w)}\right)^2}$$

This derivative is strictly positive if $\forall p \in \left(\underline{\theta}, \overline{\theta}\right)$:

$$\frac{f'(\lambda p/w)}{f(\lambda p/w)} < \frac{f'((\lambda p - 1)/w)}{f((\lambda p - 1)/w)}$$

This is true if $\ln(f(.))'' < 0$. The log-concavity of f is therefore sufficient for the monotonicity of $\xi(p)$. ■

The interpretation of the conditions is straightforward. The interior solution requires the seller's decision to change from a low to a high reserve price in $\left[\underline{\theta}, \overline{\theta}\right]$. If she observes a low market-clearing price, the conditional probability of a high control premium has to be sufficiently low to choose $\underline{\theta}$. Or, in other words, the probability of a realization x associated with a low premium has to be sufficiently high, if prices approach $\underline{\theta}$. The contrary has to hold for prices above the cut-off. Cases 1 and 2 in the proof are corner solutions. To ensure an inner cut-off point we need two additional endpoint conditions, which are presented in the next lemma.

Let us first define the following parameter for the *ex ante* profitability of a high offer in stage 2:

$$\zeta \equiv \frac{\overline{\theta} - \underline{\theta}}{\underline{\theta}}\frac{\sigma}{1-\sigma}.$$

By assumption A.1, $\zeta < 1$, that is without additional information the optimal tioli-offer is $\underline{\theta}$.

Lemma 3 *If f is log-concave and*

$$\frac{f(\lambda\underline{\theta}/w)}{f\left(\left(\lambda\underline{\theta}-1\right)/w\right)} > \zeta \tag{C.1}$$

$$\frac{f(\lambda\overline{\theta}/w)}{f\left(\left(\lambda\overline{\theta}-1\right)/w\right)} < \zeta \tag{C.2}$$

then there exists a unique cut-off p^*, implicitly defined by $\Pr(\overline{\theta}|P = p^*, p^*) = \underline{\theta}/\overline{\theta}$, in the interior of $[\underline{\theta}, \overline{\theta}]$.

Proof. In proposition 2 we have shown that log-concavity of the density function is sufficient for the existence of a unique cut-off. It lies in the interior if the posterior starts below $\underline{\theta}/\overline{\theta}$ and eventually exceeds it in the relevant interval. This is fulfilled if

$$\xi(\underline{\theta}) < \underline{\theta}/\overline{\theta} \tag{C.1'}$$

$$\xi(\overline{\theta}) > \underline{\theta}/\overline{\theta} \tag{C.2'}$$

It remains to see that conditions (C.1) and (C.2) are sufficient for (C.1') and (C.2') to be true. C1' can be rewritten as

$$\left(1 + \frac{1-\sigma}{\sigma}\frac{f(\lambda\underline{\theta}/w)}{f\left(\left(\lambda\underline{\theta}-1\right)/w\right)}\right)\frac{\underline{\theta}}{\overline{\theta}} > 1$$

Rearranging terms leads us immediately to the expression in (C.1). Note that necessary for this condition to hold is

$$1 + \frac{1-\sigma}{\sigma}\frac{f(\lambda\underline{\theta}/w)}{f\left(\left(\lambda\underline{\theta}-1\right)/w\right)} > 1$$

since $\underline{\theta}/\overline{\theta} < 1$ by assumption A.1. This is always true as long as the density is strictly positive. If we proceed in the same manner with (C.2') we get

$$\left(1 + \frac{1-\sigma}{\sigma}\frac{f(\lambda\overline{\theta}/w)}{f\left(\left(\lambda\overline{\theta}-1\right)/w\right)}\right)\frac{\underline{\theta}}{\overline{\theta}} < 1$$

which is equivalent to (C.2.). The term in brackets cannot get smaller than one while $\underline{\theta}/\overline{\theta}$ is always smaller than one. This means that the first term must be sufficiently close to one, given the realizations of $\widetilde{\theta}$. In other words, the density must have a sufficiently lower value at $x = \lambda\overline{\theta}/w$ than $x = \left(\lambda\overline{\theta}-1\right)/w$. Note that the lower σ the more difficult it becomes to meet this condition. The reverse holds for condition (C.1.). ■

Corollary 4 (of Lemma 3) *There is no density function f which ensures the existence of an interior cut-off independent of the other parameters of the model.*

Proof. Suppose that f is log-concave (i.e. the posterior for prices above p^* is monotonically increasing in p) and the parameters $\omega = (\underline{\theta}, \overline{\theta}, \sigma, w, \lambda)$ are such that the conditions of Lemma 3 are fulfilled. Then there exists a interior cut-off p^* defined by $\Pr(\overline{\theta}|P = p^*, \hat{p} = p^*) = \underline{\theta}/\overline{\theta}$. Take a second parameter vector ω' which differs from ω in the first two components such that their ratio does not change:

$$\overline{\theta}' = (1-\epsilon)p^* \text{ and } \underline{\theta}' = \frac{\underline{\theta}}{\overline{\theta}}(1-\epsilon)p^* \text{ for } \epsilon > 0$$

The indifference condition remains unchanged by this transformation of parameters and is fulfilled at a same price p^* which lies now outside the interval. Since such a transformation can be conducted for any density f, the existence of an interior solution has to depend on the parameters of the model. ■

References

[1] Admati, Anat R, 1985. "A Noisy Rational Expectations Equilibrium for Multi-asset Securities Markets," *Econometrica*, Econometric Society, 53(3), May, 629-57.

[2] Aghion, Philippe and Patrick Bolton, 1992, "An incomplete contract approach to financial contracting", *Review of Economic Studies*, 59, 473-494.

[3] Arrow, Kenneth J & M.D. Intriligator (ed.), 2000. "Handbook of Mathematical Economics," *Handbook of Mathematical Economics*, Elsevier, edition 4, volume 1

[4] Bagnoli, Mark and Ted Bergstrom, 2005. "Log-concave probability and its applications", *Economic Theory*, 26, 445-469.

[5] Barlevy, Gadi and Pietro Veronesi, 2000, "Information Acquisition in Financial Markets", *Review of Economic Studies*, 67, 79-90.

[6] Benveniste, L. and P.A. Spindt, 1989, "How Investment Bankers Determine the Offer Price and Allocation of New Issues," *Journal of Financial Economics*, 24, 343-362.

[7] Biais, Bruno, Bossaerts, P. and J.-C. Rochet, 2002, "An Optimal IPO Mechanism," *Review of Economic Studies*, 69, 117-146.

[8] Boone, Jan and Jacob K. Goeree, 2005, "Optimal Privatization Using Qualifying Auctions," CEPR Discussion Papers 5118, C.E.P.R. Discussion Papers.

[9] Dyck, Alexander and Luigi Zingales, 2004. "Private Benefits of Control: An International Comparison", *Journal of Finance*, American Finance Association, 59(2), 537-600.

[10] Grossman, Sanford J, (1981). "Nash Equilibrium and the Industrial Organization of Markets with Large Fixed Costs," *Econometrica*, Econometric Society, vol. 49(5), pages 1149-72, September.

[11] Grossman, Sanford J and Oliver D. Hart, 1980 "Takeover bids, the free-rider problem and the theory of the corporation", *The Bell Journal of Economics*, 11 (1), 42-64.

[12] Grossman, Sanford J and Joseph E Stiglitz, 1980. "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, American Economic Association, 70(3), June, 393-408.

[13] Hellwig, Martin 1980: "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory*, 22(3), 477-498.

[14] Kumaraswamy, P. 1980: "A generalized probability density function for double-bounded random processes," *Journal of Hydrology*, 46, 79–88.

[15] Pagano, Marco; Fabio Panetta & and Luigi Zingales, 1998. "Why Do Companies Go Public? An Empirical Analysis," *Journal of Finance*, American Finance Association, 53(1), 27-64.

[16] Rock, Kevin, 1986, "Why New Issues Are Underpriced," *Journal of Financial Economics*, 15, 187-212.

[17] Rubinstein, Ariel and Asher Wolinsky, 1985. "Equilibrium in a Market with Sequential Bargaining," *Econometrica*, Econometric Society, vol. 53(5), pages 1133-50, September.

[18] Zingales, Luigi, 1995, "Insider Ownership and the Decision to Go Public", *Review of Economic Studies*, 62, 425-448.

[19] Shleifer, Andrei and Lawrence H Summers, 1990. "The Noise Trader Approach to Finance," *Journal of Economic Perspectives*, American Economic Association, vol. 4(2), pages 19-33, Spring.

[20] Subrahmanyam, A. and S. Titman, 1999, "The Going-Public Decision and the Development of Financial Markets," *Journal of Finance*, 54, 1045-1082.